Page No. ___
Date

Section-1

Q4

Ans → $x^2\frac{\partial^2 z}{\partial x^2} - 4xy\frac{\partial^2 z}{\partial x \partial y} + 4y^2\frac{\partial^2 z}{\partial y^2} + 6y\frac{\partial z}{\partial y} = x^3y^4$

Put $x = e^X$

$y = e^Y$

so that $X = \log x$ & $Y = \log y$

&

let $D = \frac{\partial}{\partial X}$, $D' = \frac{\partial}{\partial Y}$ & $DD' = \frac{\partial^2}{\partial X \partial Y}$

then the given equation reduce to

$$[D(D-1) - 4DD' + 4D'(D'-1) + 6D']z = e^{3X+4Y}$$

$$\Rightarrow [(D^2 - 4DD' + 4D'^2) - (D - 2D')]z = e^{3X+4Y}$$

$$\Rightarrow (D - 2D')(D - 2D' - 1)z = e^{3X+4Y}$$

Its $CF = f_1(Y+2X) + e^{Xf_2}(Y+2X)$

$= f_1(\log y + 2\log x) + x f_2(\log y + 2\log x)$

$= f_1(\log yx^2) + x f_2(\log yx^2) = g_1(yx^2)$

$+ x g_2(yx^2)$

$$PI = \frac{1}{D - 2D' - 1}\left[\frac{1}{D - 2D'} e^{3X+4Y}\right]$$

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Page No.
Date

$$= \frac{1}{D-2D'-1}\left[\frac{1}{3-8}\int e^u du\right]$$

where $3x+4y = u$

$$= \frac{1}{D-2D'-1}\left[-\frac{1}{5}e^{3x+4y}\right]$$

$$= -\frac{1}{5}\left[\frac{1}{D-2D'-1}e^{3x+4y}\right]$$

$$= -\frac{1}{5}\left[\frac{1}{3-8-1}e^{3x+4y}\right] = \frac{1}{30}e^{3x+4y} = \frac{1}{30}x^3y^4$$

Hence the complete solution is

$$\boxed{z = C.F + P.I = g_1(y-x^2) + xg_2(y-x^2) + \frac{1}{30}x^3y^4}$$

where g_1 & g_2 are arbitrary functions.

Ans.

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