Section - 1

Q2

Ans→ Let the equation of the temperature distribution is

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{—— (1)}$$

$$u = X(x).T(t)$$

$$X \frac{\partial T}{\partial t} = c^2 T \frac{\partial^2 X}{\partial x^2}$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = \frac{1}{c^2 T}\cdot\frac{\partial T}{\partial t} = -k^2 \text{ (say)}$$

$$X = c_1 \cos kx + c_2 \sin kx$$

$$\boxed{T = c_3 . e^{-k^2 c^2 t}}$$

thus,

$$u = (A_n \cos kx + B_n \sin kx).e^{-k^2c^2t} \quad \text{—— (2)}$$

Given boundary and initial conditions are

$$\left(\frac{\partial u}{\partial x}\right)_{x=0} = 0$$

$$\left(\frac{\partial u}{\partial x}\right)_{x=a} = 0$$

Page No. (2)

$$u(x,0) = x(a-x),\ 0 < x < a$$

$$\frac{\partial u}{\partial x} = (-kA_n \sin kx + B_n k \cos kx) \cdot e^{-k^2c^2t}$$

$$0 = B_n \cdot k \cdot e^{-k^2 c^2 t}$$

$$\boxed{B_n = 0}$$

from eqn. (2), we got

$$u = A_n \cos kx \cdot e^{-k^2c^2t} \quad \text{—— (3)}$$

$$\frac{\partial u}{\partial x} = kA_n \cdot \sin kx \cdot e^{-k^2c^2t}$$

$$0 = kA_n \cdot \sin k a \cdot e^{-k^2 c^2 t}$$

$$\boxed{k = \frac{n\pi}{a}}$$

$$u = \sum_{n=0}^{\infty} A_n \cos\left(\frac{n\pi x}{a}\right) \cdot e^{-n^2\pi^2c^2 t/a^2} \quad \text{—— (4)}$$

Now at $t = 0$

$$u(x,0) = \sum A_n \cos\left(\frac{n\pi x}{a}\right)$$

$$A_n = \frac{2}{a}\int_0^a u(x,0) \cdot \cos\left(\frac{n\pi x}{a}\right) dx$$

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Page No.
Date

$$A_n = \frac{2}{a}\int_0^a x(a-x)\cos\left(\frac{n\pi x}{a}\right).\partial x$$

$$= \frac{2}{a}\Bigg[(ax-x^2)\left(\frac{a}{n\pi}.\sin\frac{n\pi x}{a}\right) - (a-2x)\left(\frac{-a^2}{n^2\pi^2}.\cos\frac{n\pi x}{a}\right) + (-2)\left(\frac{-a^3}{n^3\pi^3}.\sin\frac{n\pi x}{a}\right)\Bigg]_0^a$$

$$A_n = \frac{2}{a}\left[-(-a)\left(\frac{-a^2}{n^2\pi^2}.\cos n\pi\right) + a\left(\frac{-a^2}{n^2\pi^2}\right)\right]$$

$$= \frac{-2a^2}{n^2\pi^2}(1+\cos n\pi)$$

thus,

$$\boxed{u(x,t) = \sum_0^\infty \frac{-2a^2}{n^2\pi^2}(1+\cos n\pi).\cos\left(\frac{n\pi x}{a}\right).e^{-n^2\pi^2c^2 t/a^2}}$$

Ans.