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Section- 5

Q2
Ans. **Torsion Equation :-**

- Torsion Equation or torsion constant is defined as the geometrical property of a bar's cross-section that is involved in the axis of the bar that has a relationship b/w the angle of twist and applied torque whose SI unit is m^4.

- The torsion equation is given as follows :-

$$\boxed{\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}}$$

- **Torsion Equation derivation:-**

Following are the assumption made for the derivation of torsion equation :-

- The material is homogeneous (elastic property throughout).

- The material should follow Hook's Law.

- The material should have shear stress proportional to shear strain.

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- The cross-section area should be plane.
- The circular section should be circular.
- Every diameter of the material should rotate through the same angle.
- The stress of the material should not exceed the elastic limit.

* Consider a solid circular shaft with Radius 'R' that is subjected to a torque 'T' at one end and the other end under the same torque.

- Angle in radius $= \frac{arc}{Radius}$

$$\text{Arc } AB = R\theta = L\gamma$$

$$\gamma = \frac{R\theta}{L}$$

Where

$A \,\&\, B \rightarrow$ two fixed point on the circular shaft.

$\gamma \rightarrow$ angle subtended by AB.

$G = \frac{\tau}{\gamma}$ (modulus of rigidity)

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- Where

  τ = shear stress

  γ = shear strain

$$\frac{\tau}{G} = \Upsilon$$

$$\therefore \frac{R\theta}{L} = \frac{\tau}{G}$$

- Consider a small strip of radius with thickness dr that is subjected to shear stress.

- $\tau' \times 2\pi r dr$

  Where

  r = radius of small strip

  dr = thickness of the strip

  γ = shear stress.

$2\pi\tau' r^2 dr$ (torque at the centre of the shaft)

$$T = \int_0^R 2\pi \tau' r^2 dr \quad T = \int_0^R 2\pi \frac{G\theta}{L} r^3 dr$$

(substituting for τ')

$$T = \frac{2\pi G\theta}{L} \int_0^R r^3 dr$$

$$T = \frac{G\theta}{L}\left[\frac{\pi d^4}{32}\right]$$ (after integrating and substituting for R)

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- $\frac{G\theta}{L} J$ (substituting for the polar moment of inertia)

$$\boxed{\therefore \frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}}$$ Ans.

- Above are the steps of "Doppler' effect derivation".

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