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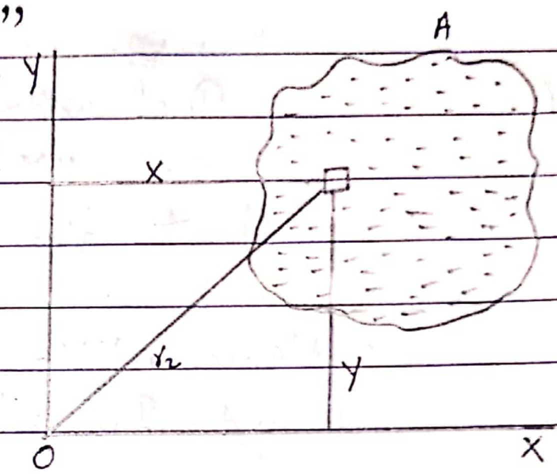
Section-"1"



Q1
Ans. -> **Product of Inertia :-**

- The product of inertia of area 'A' relative to the indicated XY regulated area is $I_{xy} = \int xy\, dA$.

- The product of inertia of ~~area A relative~~ the mass contained in volume 'V' relative to the XY area is $I_{xy} = \int xy\, \rho dv$

- Similarly for I_{yz} and I_{zx}.

- Relative to principal area of inertia, the product of inertia of a fig. is zero, If a fig. is mirror symmetrical about YZ, hence, $I_{zx} = I_{xy} = 0$

- Product of inertia of a body are measure of symmetric.

- If a particular plane is a plane of symmetry, then the product of inertia associated with any axis perpendicular to the that plane are zero,

For eg., Consider a thin laminate. The mid-plane of the laminate lies in the XY-plane that is thikness is about the plane and half is below.

- Hence, XY-plane is a plane of symmetric and $I_{xy} = I_{yz} = 0$.

x--

- Product of inertia are found other by measurement or calculation.
- Calculation are based on direct "integration", or on the "leady build-up" technique.

- In the body built-up technique, product of inertia of simple shapes are added to estimate the product of inertia of a composite shape.

(i) Consider an elemental area $r d\theta$ & thickness dr as shown in fig.

Mass of element, $dm = \rho r . d\theta . dr . t$

$= \rho t r \, d\theta . dr$

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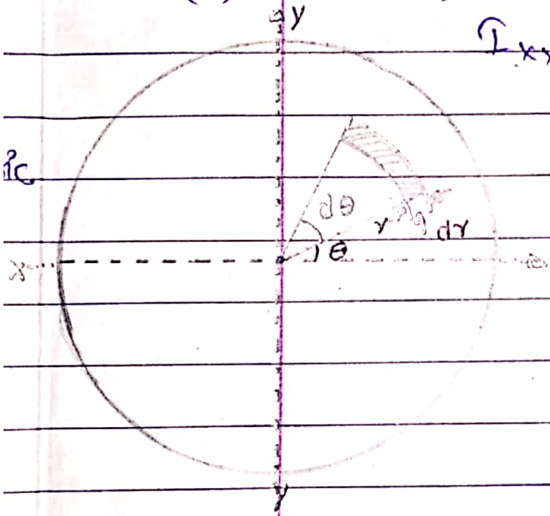
where,

p = density of the circular plate.

t = thickness of the plate.

Its distance from x-axis = $r \sin\theta$

(ii) Now,



$$I_{xx} = \oint (r\sin\theta)^2 \, dm$$
$$= \int_0^R \int_0^{2\pi} r^2 \sin^2\theta \, pt . r d\theta . dr$$
$$= pt \int_0^R \int_0^{2\pi} r^2 \left(\frac{1-\cos 2\theta}{2}\right) dr . d\theta$$
$$= pt \int_0^R \frac{r^3}{2} \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} dr$$
$$= pt \int_0^R \frac{r^3}{2} \times 2\pi r$$
$$= pt\pi \left[\frac{r^4}{4}\right]_0^R = pt \frac{\pi R^4}{4}$$

(iii) Mass of the plate, $M = p \times \pi R^2 t$

$$\boxed{I_{xx} = \frac{MR^2}{4}}$$

Similarly, $\boxed{I_{xy} = \frac{MR^2}{4}}$ Ans.

actually $I = \frac{MR^2}{4}$, is the moment of inertia of circular plate about any diametral axis in the plate.

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(iv) But

total Mass, $M = \rho t \times \pi R^2$

So, $\boxed{I_{zz} = \frac{MR^2}{2}}$ Ans.

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