∴ eq (7) 1.

Section-5

Sol- 5.2 Since R and G are negligible transmission line eq. becomes.

$$\frac{\partial e}{\partial x} = -L\frac{\partial i}{\partial t} \quad -(1)$$

and $$\frac{\partial i}{\partial x} = -C\frac{\partial e}{\partial t} \quad -(2)$$

for elimination of i, differentiating eq (1) partially w.r.t. x and eq (2) partially w.r.t. t.

$$\frac{\partial^2 e}{\partial x^2} = -L\frac{\partial^2 i}{\partial x \partial t} \text{ and } \frac{\partial^2 i}{\partial t \partial x} = C\frac{\partial^2 e}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 e}{\partial x^2} = LC\frac{\partial^2 e}{\partial t^2} \quad -(3)$$

Hence the initial conditions are $i(x,0) = i_0$, $e(x,0) = e_0 \sin\frac{\pi x}{l}$ $-(4)$

Since the ends are suddenly grounded, the boundary conditions are

$$e(0,t) = e(l,t) = 0 \quad -(5)$$

Also $i = i_0$ (constant) when $t=0$

$$\frac{\partial i}{\partial x} = 0 \text{ which gives } \frac{\partial e}{\partial t} = 0 \text{ when } t=0 \quad -(6)$$

Now let $e = XT$ be a solution of eq. (3) where X is a function of x only and T is a function of t only.

$$\frac{\partial^2 e}{\partial x^2} = X''T \text{ and } \frac{\partial^2 e}{\partial t^2} = XT''$$

∴ from eq. (2.24.3) $X''T = LCXT''$

Separating the variables $\frac{X''}{X} = LC\frac{T''}{T} = -p^2$ (say)

this leads to the ordinary differential eq.

$$\frac{d^2X}{dx^2} + p^2X = 0 \text{ and } \frac{d^2T}{dt^2} + \frac{p^2}{LC}T = 0$$

$$X = C_1\cos px + C_2\sin px$$

$$T = C_3\cos\frac{pt}{\sqrt{LC}} + C_4\sin\frac{pt}{\sqrt{LC}}$$

$$\Rightarrow e = XT = (C_1\cos px + C_2\sin px)\left(C_3\cos\frac{pt}{\sqrt{LC}} + C_4\sin\frac{pt}{\sqrt{LC}}\right) \quad -(7)$$

Applying the boundary conditions as (5) and eq (7) we get

$C_1 = 0$ and $p = \frac{n\pi}{l}$, n being an integer.

$\therefore$ eqn (7) becomes:

$$e = C_2 \sin \frac{n\pi x}{l} \left(C_3 \cos \frac{n\pi t}{l\sqrt{LC}} + C_4 \sin \frac{n\pi t}{l\sqrt{LC}} \right)$$

or

$$e = \sin \frac{n\pi x}{l} \left(A \cos \frac{n\pi t}{l\sqrt{LC}} + B \sin \frac{n\pi t}{l\sqrt{LC}} \right)$$

where $A = C_2 C_3$ and $B = C_2 C_4$ — (8)

$$\frac{\partial e}{\partial t} = \sin \frac{n\pi x}{l} \left(-\frac{An\pi}{l\sqrt{LC}} \sin \frac{n\pi t}{l\sqrt{LC}} + \frac{Bn\pi}{l\sqrt{LC}} \cos \frac{n\pi t}{l\sqrt{LC}} \right)$$

Since $\frac{\partial e}{\partial t} = 0$ when $t = 0$, we get

$B = 0$

$\therefore$ from eq (8) $e = A \sin \frac{n\pi x}{l} \cos \frac{n\pi x}{l\sqrt{LC}}$

By superposition, $e = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{l} \cos \frac{n\pi x}{l\sqrt{LC}}$ is also a solution

But $e = e_0 \sin \frac{\pi x}{l}$ when $t = 0$

$$\therefore e_0 \sin \frac{\pi x}{l} = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{l}$$

$\Rightarrow$ $A_1 = e_0$ and $A_2 = A_3 = \cdots = 0$

Hence $e = e_0 \sin \frac{\pi x}{l} \cos \frac{\pi t}{l\sqrt{LC}}$

Now, $-L \frac{\partial i}{\partial t} = \frac{\partial e}{\partial x}$,

$$\frac{\partial i}{\partial t} = \frac{-1}{L} \cdot \frac{e_0 \pi}{l} \cos \frac{\pi x}{l} \cos \frac{\pi t}{l\sqrt{LC}}$$

Integrating w.r.t. t regarding x as constant

$$i = -\frac{e_0 \pi}{Ll} \cos \frac{\pi x}{l} \cdot \frac{l\sqrt{LC}}{\pi} \sin \frac{\pi t}{l\sqrt{LC}} + f(x) \quad (9)$$

where $f(x)$ is an arbitrary constant function

Since $i = i_0$ when $t = 0$, we have $i_0 = 0 + f(x)$ or $f(x) = i_0$

$\therefore$ from eq (9) we have.

$$i = i_0 - e_0 \sqrt{\frac{C}{L}} \cos \frac{\pi x}{l} \sin \frac{\pi t}{l\sqrt{LC}}.$$