Sol-3 (a) Here Lagrange's subsidiary eqⁿ are

$$\frac{dx}{x^2-yz} = \frac{dy}{y^2-zx} = \frac{dz}{z^2-xy}$$

$$\therefore \frac{dx-dy}{(x-y)(x+y+z)} = \frac{dy-dz}{(y-z)(x+y+z)} = \frac{dz-dx}{(z-x)(x+y+z)}$$

taking the first two members we have,

$$\frac{dx-dy}{x-y} = \frac{dy-dz}{y-z}$$

which on integration gives

$$\log(x-y) = \log(y-z) + \log a$$

or $\log\left(\frac{x-y}{y-z}\right) = \log a$ or $\frac{x-y}{y-z} = a$ — (1)

Similarly taking the last two members —

$$\frac{y-z}{z-x} = b$$

from eqⁿ (1) and eqⁿ (2) the general solution

... possible solution

$$\phi\left[\frac{x-y}{y-z}, \frac{y-z}{z-x}\right] = 0.$$

(b) Rewriting the given eqⁿ as

$$y^2 z p + x^2 z q = y^2 x$$

the subsidiary eqⁿ are

$$\frac{dx}{y^2 z} = \frac{dy}{x^2 z} = \frac{dz}{y^2 x}$$

the first two fractions give $x^2 dx = y^2 dy$.

Integrating we get $x^3 - y^3 = a$ (1)

Again the first and third fraction are

$$x\,dx = z\,dz$$

Integrating we get $x^2 - z^2 = b$ — (2)

Hence from eqⁿ (1) and (2) the complete solution is

$$x^3 - y^3 = f(x^2 - z^2)$$

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