Session - 2

Sol - 2 Given $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Let, $u = XY$

$$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} = 0$$

$$\Rightarrow \frac{1}{X} \frac{\partial^2 X}{\partial x^2} = -\frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = -k^2 \text{ (say)}$$

$$\Rightarrow X = C_1 \cos kx + C_2 \sin kx, \quad Y = C_3 e^{ky} + C_4 e^{-ky}$$

$$u = (C_1 \cos kx + C_2 \sin kx)(C_3 e^{ky} + C_4 e^{-ky}) \quad (*)$$

$\Rightarrow u(0, y) = 0$ — (1)

$$0 = C_1 (C_3 e^{ky} + C_4 e^{-ky})$$

$$C_1 = 0$$

from eq (1)

$$u = \sin kx \, (A_n e^{ky} + B_n e^{-ky}) \quad (2)$$

$$u(\pi, y) = 0$$

$$\sin k\pi = 0 \Rightarrow k = n$$

$$u = \sin nx \, (A_n e^{ny} + B_n e^{-ny}) \quad (3)$$

(iii) $\lim_{y \to \infty} u(x,y) = 0$. It is true only when $A_n = 0$.

from eq. (3)

$$u = \sum B_n e^{-ny} \sin nx \quad (4)$$

Now — $u(x, 0) = u_0$

$$u_0 = \sum B_n \sin nx$$

$$B_n = \frac{2}{\pi} \int_0^{\pi} u_0 \sin nx \, dx = \frac{2u_0}{\pi} \left[\frac{-\cos nx}{n} \right]_0^{\pi} = \frac{2u_0}{\pi} \left(\frac{1 - (-1)^n}{n} \right)$$

thus from eq. (4)

$$u = \sum \frac{2u_0}{\pi n} \left(1 - (-1)^n \right) e^{-ny} \sin nx$$

Ans