Section - 2

Sol1 ① i) Let $x = e^X$, $y = e^Y$ so that $X = \log x$ and $Y = \log y$ and let $D = \frac{\partial}{\partial X}$ and $D' = \frac{\partial}{\partial Y}$,

then the given eq. reduces to

$$[D(D-1) - D'(D'-1) + D - D']z = X$$

$$\Rightarrow \quad (D^2 - D'^2]z = X$$

which is a homogeneous linear partial differential eq. with constant coefficients.

$$CF = \phi_1(Y+X) + \phi_2(Y-X)$$

and $PI = \frac{1}{D^2 - D'^2}(X) = \frac{1}{(1)^2 - (0)^2}\iint u\, du\, du$

where $X = u$

$$= \int \frac{u^2}{2} du = \frac{u^3}{6} = \frac{X^3}{6}$$

Hence solution to eq. (1) is

$$z = \phi_1(Y+X) + \phi_2(Y-X) + \frac{X^3}{6}$$

$$= \phi_1(\log y + \log x) + \phi_2(\log y - \log x) + \frac{(\log x)^3}{6}$$

therefore the complete solution to the given differential eq. is.

$$z = \phi_1(\log xy) + \phi_2\left(\log \frac{y}{x}\right) + \frac{1}{6}(\log x)^3$$

$$z = f_1(xy) + f_2\left(\frac{y}{x}\right) + \frac{1}{6}(\log x)^3$$

where $f_1$ and $f_2$ are arbitrary function