Q. 2
Ans →

SECTION - 2

Given = $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Let, $u = xy$

$$y\frac{\partial^2 Y}{\partial x^2} + x\frac{\partial^2 Y}{\partial Y^2} = 0$$

$$\frac{1}{x}\frac{\partial^2 Y}{\partial x^2} + x\frac{\partial^2 Y}{\partial Y^2} = 0$$

$$\frac{1}{x}\frac{\partial^2 X}{\partial x^2} = -\frac{1}{Y}\frac{\partial^1 Y}{\partial Y^2} = -k^2 (say)$$

$$x = C_1 \cos ks + C_2 \sin kx, \; Y = C_3 e^{kY} + C_4 e^{-ky}$$

$$u = (C_1 \cos ks + C_2 \sin kx)(c_3 e^{kY} + c_4 e^{-kY})$$

$$u(0,Y) = 0 \quad — (1)$$

$$0 = C_1 (c_3 e^{kY} + c_4 e^{-kY})$$

$$C_1 = 0$$

From eq (1)

$$U = \sin kx (A_n e^{kY} + B_n e^{-ky}) \quad — (2)$$

$$u(\pi, y) = 0$$

$$\sin k\pi = 0 \Rightarrow k = n$$

$$u = \sin nx (A_n e^{nY} + B_n e^{-ny}) \quad — (3)$$

$\lim_{Y\to\infty} u(x,y) = 0$ it satisfies only when

$$A_n = 0$$

From eq - (3)

$$u = \sum B_n e^{-ny} \sin nx \quad — (4)$$

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Now, $u(x,0) = u_0$

$$u_0 = \sum B_n \sin nx$$

$$B_n = \frac{2}{\pi}\int_0^{\pi} u_0 \sin nx \, dx = \frac{-2u_0}{\pi}\left[\frac{\cos nx}{n}\right]_0^{\pi} = \frac{-2u_0}{\pi}\left[\frac{(-1)^n - 1}{n}\right]$$

Thus from eq (4)

$$u = \sum \frac{-2u_0}{\pi n}\left[(-1)^n - 1\right] e^{-ny} \sin nx .$$