2. Since R and G are negligible transmission line eq. becomes.

$$\frac{\partial e}{\partial x} = -L\frac{\partial i}{\partial t} \quad -(1)$$

$$\frac{\partial i}{\partial x} = -C\frac{\partial e}{\partial t} \quad -(2)$$

From eq (1) & (2).

$$\frac{\partial^2 e}{\partial x^2} = L\frac{\partial^2 i}{\partial x \partial t} \quad \text{and} \quad \frac{\partial^2 i}{\partial t \partial x} = C\frac{\partial^2 e}{\partial t^2}$$

Hence $$\frac{\partial^2 e}{\partial x^2} = LC\frac{\partial^2 e}{\partial t^2} \quad -(3)$$

The initial condition are $i(x, 0) = i_0$, $e(x, 0) = e_0 \sin\frac{\pi x}{l}$ $-(4)$

$$e(0,t) = e(l,t) = 0 \quad -(5)$$

Also $i = i_0$ (constant) when $t = 0$

$$\frac{\partial i}{\partial x} = 0 \text{ which gives } \frac{\partial e}{\partial t} = 0 \text{ when } t = 0 \quad -(6)$$

$$\frac{\partial^2 e}{\partial x^2} = X''T \quad \& \quad \frac{\partial^2 e}{\partial t^2} = XT''$$

From eq. (3) $X''T = LCXT''$

Separating the variables $\frac{X''}{X} = LC\frac{T''}{T} = -p^2$ (say)

$$\frac{d^2X}{dx^2} + p^2 X = 0 \;\&\; \frac{d^2T}{dt^2} + \frac{p^2}{LC}T = 0$$

$$X = c_1 \cos px + c_2 \sin px$$

$$T = c_3 \cos \frac{pt}{\sqrt{LC}} + c_4 \sin \frac{pt}{\sqrt{LC}}$$

$$e = XT = (c_1 \cos px + c_2 \sin px)$$

$$\left(c_3 \cos \frac{pt}{\sqrt{LC}} + c_4 \sin \frac{pt}{\sqrt{LC}}\right) \quad - (7)$$

from eq (5) & (7)

$c_1 = 0$ & $p = \frac{n\pi}{l}$, n being an integer

eq (7) becomes

$$e = c_2 \sin \frac{n\pi x}{l}\left(c_3 \cos \frac{n\pi t}{l\sqrt{LC}} + \cancel{B}c_4 \sin \frac{n\pi t}{l\sqrt{LC}}\right)$$

$$e = \sin \frac{n\pi x}{l}\left(A \cos \frac{n\pi t}{l\sqrt{LC}} + B \sin \frac{n\pi t}{l\,LC}\right)$$

$$A = c_2 c_3 \;\&\; B = c_2 c_4 \quad \text{— } (8)$$

$$\frac{\partial e}{\partial t} \sin \frac{n\pi x}{t}\left(-\frac{An\pi}{l\sqrt{LC}} \sin \frac{n\pi t}{l\sqrt{LC}} + \frac{Bn\pi}{l\sqrt{LC}} \cos \frac{n\pi t}{l\sqrt{LC}}\right)$$

since $\frac{\partial e}{\partial t} = 0$ when $t = 0$, we get

$$B = 0$$

from eq (8) $e = A \sin \frac{n\pi x}{l} \cos \frac{n\pi x}{l\sqrt{LC}}$

$$A_1 = e_0 \;\&\; A_2 = A_3 = 0 \cdots\cdots = 0$$

Hence, $e = e_0 \sin \frac{\pi x}{l} \cos \frac{\pi t}{l\sqrt{LC}}$

Integrating w.r.t. t, regarding x as constant

$$i = \frac{e_0 \pi}{2s} \cos \frac{\pi x}{s} \cdot \frac{s\sqrt{LC}}{\pi} \sin \frac{\pi t}{s\sqrt{LC}} + f(x) \quad (9)$$

where $f(x)$ is an arbitrary constatant. function.

Since $i = i_0$ when $t=0$, we have $i_0 = 0 + f(x)$ or

$$f(x) = i_0$$

$\therefore$ from eq (9)

$$i = i_0 - e_0 \sqrt{\frac{C}{L}} \cos \frac{\pi}{l} \sin \frac{\pi t}{l\sqrt{LC}}$$