Q-1 Ans

SECTION - 5

Laplace equation in two dimension is given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{— (1)}$$

Let $u = XY$ — (2)

Then, $\frac{\partial^2 u}{\partial x^2} = X''Y$

$$\frac{\partial^2 u}{\partial y^2} = XY''$$

Substituting in eq (1)

$$X''Y + XY'' 0$$

$$\frac{X''}{X} = -\frac{Y''}{Y} = K \text{ (say)} \quad \text{— (3)}$$

Now, $\frac{d^2 X}{dx^2} - KY = 0$ — (4)

Solving eq (4)

i) when K is positive and $K = P^2$

$$X = c_1 e^{Px} + c_2 e^{-Px}$$

$$Y = c_3 \cos Py + c_4 \sin Py$$

ii) when K is negative and $K = -P^2$

$$X = c_1 \cos Px + c_2 \sin Px, \; Y = c_3 e^{Py} + c_4 e^{-Py}$$

iii) when $K = 0$

$$X = c_1 x + c_2 Y = c_3 y + c_4$$

Thus, the various possible solutions of Laplace eq (2)

$$U = (C_1 e^{px} + C_2 e^{-px})(C_3 \cos py + C_4 \sin py) \quad -(5)$$

$$U = (C_1 \cos px + C_2 \sin px)(C_3 e^{py} + C_4 e^{-py}) \quad -(6)$$

$$U = (C_1 x + C_2)(C_3 y + C_4) \quad -(7)$$