SECTION - 3

1.) $f(x) \geq 0$ for every x in $(1,2)$ and

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{0} e^{-x}\,dx = 1$$

Hence the function $f(x)$ satisfies the requirements for a density function

ii) Required Probability $= P(1 \leq x \leq 2) = \int_{1}^{2} e^{-x}\,dx = e^{-1} - e^{-2}$

$$= 0.368 - 0.135 = 0.233$$

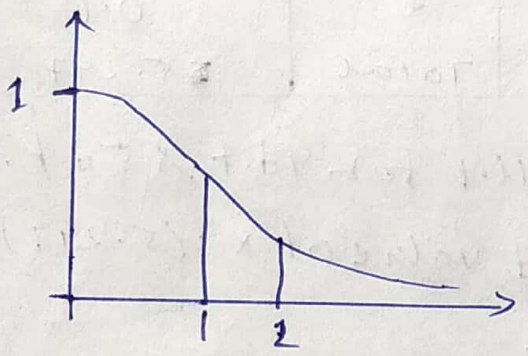
This Probability is equal to the shaded area in fig (a).

iii) Cumulative probability function $P(2)$!

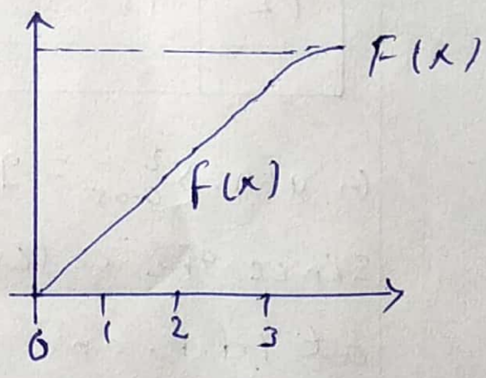
$$\int_{-\infty}^{2} f(x)\,dx = \int_{-x}^{0} 0\,dx + \int_{0}^{2} e^{-x}\,dx = 1 - e^{-2}$$

$$= 1 - 0.135 = 0.865$$

Which is shown if fig (b).



(a)



(b)

Probability of female birth $= p = \frac{1}{2}$

Probability of male birth $= \frac{1}{2}$

$(q+p)^n = pq^n + {}^nC_1\, pq^{n-1} + {}^nC_2\, p^2q^{n-2} + {}^nC_3\, p^3q^{n-3} \ldots p^n$

$= \left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 + 6\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^0$

No. of girls $= 240\left[\frac{1}{16} + \frac{4}{16} + \frac{6}{16} + \frac{4}{16} + \frac{1}{16}\right]$

$= 240 \times \frac{1}{16} + 240 \times \frac{4}{16} + 240 \times \frac{6}{16} + 240 \times \frac{4}{16} + 240 \times \frac{1}{16}$

$= 15 + 60 + 90 + 60 + 15$

These are the expected frequencies of female births.

O	E	O − E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
10	15	−5	25	1.67
55	60	−5	25	0.41
105	90	15	225	2.5
58	60	−2	4	0.067
12	15	−3	9	0.6
			Total	5.247

Given, $\chi^2_{0.05} = 9.49$ & 11.1 for 4 d.f & 5 d.f.

Since the calculated value of χ^2 $(5.247) < \chi^2$ value at 4 d.f. and 5 d.f.