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Sub - Mathematics - II
BRANCH - EEE 2nd YEAR

(11)

Q.1
1. Ans

SECTION - 1

Given Laplace equtionis

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{— (1)}$$

Let $u = XY$, where X is a function of x and Y is a function of y only.

$$\frac{\partial^2 u}{\partial x^2} = Y \frac{\partial^2 X}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial y^2} = X \frac{\partial^2 Y}{\partial y^2}$$

From eq (2)

$$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} = 0$$

$$-\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2}$$

case i - $-\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = 0 \text{ (say)}$

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = 0$$

$$\frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = 0$$

$$X = C_1 x + C_2, \quad Y = C_3 y + C_4$$

$$y = 0, \; Y = 0 \Rightarrow C_4 = 0$$

$$y = b, \; Y = 0 \Rightarrow C_3 = 0$$

$$Y = 0$$

$$u = XY = X(0)$$

$u = 0$ (Not Possible).

Case ii :- $-\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = k^2$ (say)

$$\frac{\partial^2 Y}{\partial y^2} - k^2 Y = 0$$

$X = C_1 \cos kx + C_2 \sin kx$, $Y = C_3 e^{ky} + C_4 e^{-ky}$

$y = 0, Y = 0$

$C_3 + C_4 = 0$

$C_4 = -C_3$

$Y = 0$ at $y = b$

$0 = C_3 e^{kb} - C_3 e^{-kb}$

$C_3 (e^{kb} - e^{-kb}) = 0$

$C_3 = 0, C_4 = 0, Y = 0$

Case iii) $-\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k^2$ (say)

$X = C_1 e^{kx} + C_2 e^{-kx}$

$Y = C_3 \cos ky + C_4 \sin ky$

$y = 0, Y = 0, C_3 = 0$

$Y = C_4 \sin ky$

$y = b, Y = 0$

$0 = C_4 \sin kb$

$\sin kb = 0$

$kb = n\pi$

$$k = \frac{n\pi}{b}$$

$$u = (C_1 e^{kx} + C_2 e^{-kx}) C_4 \sin \frac{n\pi y}{b} \quad \text{(2)}$$

$x = 0, u = 0$

$$0 = (c_1 + c_2)\, c_4 \sin \frac{n\pi y}{b}$$

$$c_1 + c_2 = 0$$

$$c_2 = -c_1$$

From eq (2)

$$u = \frac{2}{2} c_4 c_1 \left(e^{kx} - e^{-kx}\right) \sin \frac{n\pi y}{b}$$

$$u = \sum_{n=0}^{\infty} b_n \left(\frac{e^{\frac{n\pi x}{b}} - e^{-\frac{n\pi x}{b}}}{2} \right) \sin\left(\frac{n\pi y}{b}\right) \quad \text{— (3)}$$

$$b_n = 2c_1 c_4$$

$$x = a,\ u = f(y)$$

From eq(3) $\cancel{f(x)} = f(y) = \sum_{n=0}^{\infty} b_n \left(\frac{e^{\frac{n\pi a}{b}} - e^{-\frac{n\pi a}{b}}}{2} \right) \sin\left(\frac{n\pi y}{b}\right)$

$$f(y) = \sum_{n=0}^{\infty} b_n \sinh\left(\frac{n\pi a}{b}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$b_n \sinh\left(\frac{n\pi a}{b}\right) = \frac{2}{b} \int_0^b f(y) \sin\left(\frac{n\pi y}{b}\right) dy$$

$$b_n = \frac{2}{b \sinh\left(\frac{n\pi a}{b}\right)} \int_0^b f(y) \sin\left(\frac{n\pi y}{b}\right) dy \quad \text{— (4)}$$

$$u = \sum_{n=0}^{\infty} b_n \sinh\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

Ans