Section-02

Q.2

Ans

Given, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Let $u = XY$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k^2$$

$$X = c_1 \cos kx + c_2 \sin kx$$
$$Y = c_3 e^{ky} + c_4 e^{-ky}$$
$$u = (c_1 \cos kx + c_2 \sin kx)(c_3 e^{ky} + c_4 e^{-ky}) \quad \text{— eq}^n \ (1)$$

$u(0, y) = 0$

$$0 = c_1 (c_3 e^{ky} + c_4 e^{-ky})$$
$$c_1 = 0$$

from eq^n – (1)

$$u = \sin kx\,(A_n e^{ky} + B_n e^{-ky}) \quad \text{eq}^n \ (2)$$

$u(\pi, y) = 0$

$$\sin k\pi = 0$$
$$k = n$$
$$u = \sin nx\,(A_n e^{ny} + B_n e^{-ny})$$
– eq^n (3)

$\lim_{y \to \infty} u(x, y) = 0$, it statisfies only when $A_n = 0$

from eqn-3

$$u = \sum B_n e^{-ny} \sin x \quad - \text{eq } 4$$

$$u(x, 0) = u_0$$

$$u_0 = \sum B_n \sin nx$$

$$B_n = \frac{2}{\pi} \int_0^{\pi} u_0 \sin nx \, dx = \frac{-2u_0}{\pi} \left[\frac{\cos nx}{n} \right]_0^{\pi}$$

$$= \frac{-2u_0}{\pi} \left[\frac{(-1)^n - 1}{n} \right]$$

from eqn-4

$$u = \sum \frac{-2u_0}{\pi n} \left[(-1)^n - 1 \right] e^{-ny} \sin nx$$

Ans