Q.2

Ans. Since R and G are negligible, transmission line equations becomes.

$$\frac{\partial e}{\partial x} = -L\frac{\partial i}{\partial t} \quad \text{— eq}^n \text{ (1)}$$

$$\frac{\partial i}{\partial x} = -C\frac{\partial e}{\partial t} \quad \text{eq}^n\text{-(2)}$$

for elimination of i, differentiating eq^n-(1) w.r.t x and eq-(2) partially w.r.t. t,

$$\frac{\partial^2 e}{\partial x^2} = -L\frac{\partial^2 i}{\partial x \partial t} \text{ and } \frac{\partial^2 i}{\partial t \partial x} = C\frac{\partial^2 e}{\partial t^2}$$

Hence, $$\frac{\partial^2 e}{\partial x^2} = LC\frac{\partial^2 e}{\partial t^2} \quad \text{eq}^n\text{-(3)}$$

The initial conditions are $i(x,0) = i_0$,

$$e(x,0) = e_0 \sin\frac{\pi x}{l} \quad \text{— eq}^n \text{ (4)}$$

since, the ends are suddenly grounded, the boundary conditions are

$$e(0,t) = e(l,t) = 0 \quad \text{— eq}^n \text{ (5)}$$

Also $i = i_0$ (constant) when $t = 0$

$$\frac{\partial i}{\partial x} = 0 \text{ which gives } \frac{\partial e}{\partial t} = 0$$

when $t = 0$ — eq^n-(6)

Now let $e = XT$ be sol. of eq^n-(3)

$$\frac{\partial^2 e}{\partial x^2} = X''T \text{ and } \frac{\partial^2 e}{\partial t^2} = XT''$$

from eq^n-(3) $X''T = LCXT''$

Separating the variables $\frac{X''}{X} = LC\frac{T''}{T} = -p^2$

this leads to the ordinary differential equations.

$$\frac{d^2X}{dx^2} + p^2X = 0 \text{ and } \frac{d^2T}{dt^2} + \frac{p^2}{LC}T = 0$$

$$X = C_1 \cos px + C_2 \sin px$$

$$T = C_3 \cos\frac{pt}{\sqrt{LC}} + C_4 \sin\frac{pt}{\sqrt{LC}}$$

$$e = XT = (C_1 \cos px + C_2 \sin px)$$

$$\left(C_3 \cos\frac{pt}{\sqrt{LC}} + C_4 \sin\frac{pt}{\sqrt{LC}}\right) \quad - \text{eq}^n \ 7$$

applying the boundary condition eq^n 5 in eq^n 7

$C_1 = 0$ and $p = \frac{n\pi}{l}$, n being an integer.

$$e = C_2 \sin\frac{n\pi x}{l}\left(C_3 \cos\frac{n\pi t}{l\sqrt{LC}} + C_4 \sin\frac{n\pi t}{l\sqrt{LC}}\right)$$

$$e = \sin\frac{n\pi x}{l}\left(A \cos\frac{n\pi t}{l\sqrt{LC}} + B \sin\frac{n\pi t}{l\sqrt{LC}}\right)$$

$- \text{eq}^n \ 8$

where $A = C_2C_3$ and $B = C_2C_4$

$$\frac{\partial e}{\partial t} = \sin\frac{n\pi x}{l}\left(-\frac{An\pi}{l\sqrt{LC}} \sin\frac{n\pi t}{l\sqrt{LC}} + \frac{Bn\pi}{l\sqrt{LC}} \cos\frac{n\pi t}{l\sqrt{LC}}\right)$$

Since $\frac{\partial e}{\partial t} = 0$ when $t = 0$, we get

$$B = 0$$

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from eq^n - (8)

$$e = A \sin\frac{n\pi x}{l} \cos\frac{n\pi x}{l\sqrt{LC}}$$

$$e = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{l} \cos\frac{n\pi x}{l\sqrt{C}}$$

is also a solution.

But $\quad e = e_0 \sin\frac{\pi x}{l}$ when $t=0$

$$e_0 \sin\frac{\pi x}{l} = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{l}$$

$$A_1 = e_0 \text{ and } A_2 = A_3 = \dots = 0$$

Hence, $\quad e = e_0 \sin\frac{\pi x}{l} \cos\frac{\pi t}{l\sqrt{LC}}$

Now, $\quad -L\frac{\partial i}{\partial t} = \frac{\partial e}{\partial x}$

$$\frac{\partial i}{\partial t} = -\frac{1}{L}\cdot\frac{e_0\pi}{l} \cos\frac{\pi x}{l} \cos\frac{\pi t}{l\sqrt{LC}}$$

integrating w.r.t t,

$$i = -\frac{e_0\pi}{Ll} \cos\frac{\pi x}{l}\cdot\frac{l\sqrt{LC}}{\pi} \sin\frac{\pi t}{l\sqrt{LC}} + f(x)$$

- eq^n (9)

where $f(x)$ is an arbitrary Constan function

from eq^n - (9) we have

$$i = i_0 - e_0\sqrt{\frac{C}{L}} \cos\frac{\pi x}{l} \sin\frac{\pi t}{l\sqrt{LC}}$$ Ans