Section-05

Q.1

Ans Laplace eqn in two dimension is given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{— eq}^n \text{ (1)}$$

$$u = XY \quad \text{eq}^n \text{ (2)}$$

$$\frac{\partial^2 u}{\partial x^2} = X''Y$$

$$\frac{\partial^2 u}{\partial y^2} = XY''$$

Substituting in eqn-(1)

$$X''Y + XY'' = 0$$

$$\frac{X''}{X} = \frac{Y''}{Y} = K \quad \text{eq}^n\text{-(3)}$$

$$\frac{d^2X}{dx^2} - KX = 0 \quad \text{eq}^n\text{-(4)}$$

$$\frac{d^2Y}{dy^2} + KY = 0$$

Solving eqn-(4) we get

when K is positive and $K = p^2$

$$X = C_1 e^{px} + C_2 e^{-px}$$

$$Y = C_3 \cos py + C_4 \sin py$$

when K is negative and $K = -p^2$

$$X = C_1 \cos px + C_2 \sin px,$$

$$Y = C_3 e^{py} + C_4 e^{-py}$$

when $k = 0$

$$X = c_1 x + c_2$$

$$Y = c_3 y + c_4$$

thus, the various possible solutions of Laplace eqn are

$$u = (c_1 e^{px} + c_1 e^{-px})(c_3 \cos py + c_4 \sin py) \quad \text{- eqn } 5$$

$$u = (c_1 \cos px + c_2 \sin px)(c_3 e^{py} + c_4 e^{-py}) \quad \text{- eqn } 6$$

$$u = (c_1 x + c_2)(c_3 y + c_4) \quad \text{- eqn } 7$$

Ans

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