Ans:-

Put

$x = e^X$

$y = e^Y$

$X = \log x$

$Y = \log y$

and Let

$D \equiv \frac{\partial}{\partial X}$

$D' \equiv \frac{\partial}{\partial Y}$ and

$DD' = \frac{\partial^2}{\partial X \partial Y}$ the the given eqn reduces to

$$[D(D-1) - 4DD' + 4D'(D'-1) + 6D']z = e^{3X+4Y}$$

$$\Rightarrow [(D^2 - 4DD' + 4D'^2) - (D - 2D')]z = e^{3X+4Y}$$

$$\Rightarrow (D-2D')(D-2D'-1)z = e^{3X+4Y}$$

$$CF = f_1(Y+2X) + e^X f_2(Y+2X)$$

$$= f_1(\log y + 2\log x) + x f_2(\log y + 2\log x)$$

$$= f_1(\log y x^2) + x f_2(\log y x^2)$$

$$= g_1(y x^2) + x g_2(y x)^2$$

$$PI = \frac{1}{D \quad 2D' - 1}\left[\frac{1}{3-8}\int e^u \, du\right]$$ where $3X+4Y = u$

$$= \frac{1}{D-2D'-1}\left[-\frac{1}{5}e^{3x+4y}\right]$$

$$= -\frac{1}{5}\left[\frac{1}{D-2D'-1}e^{3x+4y}\right]$$

$$= -\frac{1}{5}\left[\frac{1}{3-8-1}e^{3x+4y}\right]$$

$$= \frac{1}{30}e^{3x+4y}$$

$$= \frac{1}{30}x^3+y^4$$

Hence the Complete Solution is :-

$$z = CF + PI$$

$$= g_1(yx^2) + xg_2(yx^2) + \frac{1}{30}x^3y^4$$

where g_1 and g_2 are arbitrary functions.