Q.3

Ans One dimensional wave eqn is given by

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{eq}^n - ①$$

where

$$c^2 = \frac{T}{m}$$

T = Tension in the string

m = mass per unit length of the string.

let $\quad u = X(x)\,T(t) \quad - \text{eq}^n$ ②

where X is a function of x only and T is a function of t only.

differentiating eqn ⑪ partially w.r.t x and t and putting value in eqn - ①

$$\frac{\partial^2 u}{\partial t^2} = X \frac{\partial^2 T}{\partial t^2}$$

$$\frac{\partial^2 u}{\partial x^2} = T \frac{\partial^2 X}{\partial x^2}$$

From one dimensional wave eqn

$$X \frac{\partial^2 T}{\partial t^2} = c^2 T \frac{\partial^2 X}{\partial x^2}$$

Separating the variables

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = \frac{1}{c^2 T}\frac{\partial^2 T}{\partial t^2} = -k^2 \text{ or } k^2 \text{ or } (0)$$

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Case-1 :

when $\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -k^2$

and $\frac{1}{C^2 T}\frac{\partial^2 T}{\partial t^2} = -k^2$

or $\frac{\partial^2 X}{\partial x^2} + k^2 X = 0$ and $\frac{\partial^2 T}{\partial t^2} + k^2 C^2 T = 0$

or $(D^2 + k^2)X = 0$ and $(D^2 + k^2C^2)T = 0$

Auxiliar eqn are

$m^2 + k^2 = 0$ and

$m^2 + k^2C^2 = 0$

$m = \pm ki$ and $m = \pm kci$

the complementary functions are

$X = C_1 \cos kx + C_2 \sin kx$

$T = C_3 \cos kct + C_4 \sin kct$

$u = XT$

$u = (C_1 \cos kx + C_2 \sin kx)(C_3 \cos kct + C_4 \sin kct)$ — eqn 3

Case-2 :-

$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = k^2$ and $\frac{1}{C^2 T}\frac{\partial^2 T}{\partial t^2} = k^2$

$m = k^2$ and $m^2 = k^2C^2$

$m = \pm k$ and $m = \pm kC$

$X = C_5 e^{kx} + C_6 e^{-kx}$

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and $T = C_7 e^{kct} + C_8 e^{-kct}$

$$u = (C_5 e^{kx} + C_6 e^{-kx})(C_7 e^{kct} + C_8 e^{-kct}) \quad \text{- eq}^n \text{ (4)}$$

Case 3 :-

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = 0 \text{ and } \frac{1}{C^2 T}\frac{\partial^2 T}{\partial t^2} = 0$$

$m = 0, 0$ and $m = 0, 0$

$X = C_9 + C_{10} x$ and

$T = C_{11} + C_{12} t$

$$u = (C_9 + C_{10} x)(C_{11} + C_{12} t) \quad \text{eq}^n \text{ - 5}$$

Ans

the solution given by eq^n - (3) satisfies the one dimensional wave eq^n.

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