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Section-01

Q.1

Ans Given Laplace equation is :-

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad - eq^n - ①$$

Let $u = XY$, where X is a function of x only and Y is a function of y only.

$$\frac{\partial^2 u}{\partial x^2} = Y \frac{\partial^2 X}{\partial x^2}$$

and

$$\frac{\partial^2 u}{\partial y^2} = X \frac{\partial^2 Y}{\partial y^2}$$

from $eq^n - 2$.

$$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} = 0$$

$$-\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2}$$

Case i :- $-\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = 0$

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = 0$$

and $$\frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = 0$$

$$X = C_1 x + C_2, \quad Y = C_3 y + C_4$$

At $y = 0,\ Y = 0 \Rightarrow C_4 = 0$

$y = b,\ Y = 0 \Rightarrow C_3 = 0$

$Y = 0$

Thus, $u = XY = X(0)$

$u = 0$ (not possible)

Case ii :-

$$-\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = k^2$$

$$\frac{\partial^2 X}{\partial x^2} + k^2 X = 0$$

$$\frac{\partial^2 Y}{\partial y^2} - k^2 Y = 0$$

$X = C_1 \cos kx + C_2 \sin kx,$

$Y = C_3 e^{ky} + C_4 e^{-ky}$

if $y = 0,\ Y = 0$

$C_3 + C_4 = 0$

$C_4 = -C_3$

$Y = 0$ at $y = b$

$0 = C_3 e^{kb} - C_3 e^{-kb}$

$C_3(e^{kb} - e^{-kb}) = 0$

$C_3 = 0,$

$C_4 = 0$ (Not possible)

$Y = 0$

Case iii :-
$$-\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k^2$$

$$X = c_1 e^{kx} + c_2 e^{-kx}$$

$$Y = c_3 \cos ky + c_4 \sin ky$$

$$y = 0,\ Y = 0,\ c_3 = 0$$

$$Y = c_4 \sin ky$$

$$y = b,\ Y = 0$$

$$0 = c_4 \sin kb$$

$$\sin kb = 0$$

$$kb = n\pi$$

$$k = \frac{n\pi}{b}$$

thus,
$$u = (c_1 e^{kx} + c_2 e^{-kx}) c_4 \sin\frac{n\pi y}{b} \quad - \text{eq}^n \text{ (2)}$$

$$x = 0,\ u = 0$$

$$0 = (c_1 + c_2) c_4 \sin\frac{n\pi y}{b}$$

$$c_1 + c_2 = 0$$

$$c_2 = -c_1$$

from eq^n - (2)

$$u = \frac{2}{2} c_4 c_1 (e^{kx} - e^{-kx}) \sin\frac{n\pi y}{b}$$

$$u = \sum_{n=0}^{\infty} b_n \left(\frac{e^{\frac{n\pi x}{b}} - e^{\frac{-n\pi x}{b}}}{2}\right) \sin\left(\frac{n\pi y}{b}\right) \quad \text{eq}^n \text{ (3)}$$

let
$$b_n = 2 c_1 c_4$$

$$x = a,\ u = f(y)$$

From eqn - 3

$$f(y) = \sum_{n=0}^{\infty} bn \left[\frac{e^{\frac{n\pi a}{b}} - e^{\frac{-n\pi a}{b}}}{2} \right] \sin\left(\frac{n\pi y}{b}\right)$$

$$F(y) = \sum_{n=0}^{\infty} bn \sinh\left(\frac{n\pi a}{b}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$b_n \sinh\left(\frac{n\pi a}{b}\right) = \frac{2}{b} \int_0^b f(y) \sin\left(\frac{n\pi y}{b}\right) dy$$

$$bn = \frac{2}{b \sinh\left(\frac{n\pi a}{b}\right)} \int_0^b f(y) \sin\left(\frac{n\pi y}{b}\right) dy \quad \text{eq}^n\text{-}4$$

$$u = \sum_{n=0}^{\infty} bn \sinh\left(\frac{n\pi x}{b}\right) \sin\left(\frac{n\pi y}{b}\right)$$

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