

Q.3

Ans:- Expression for reluctance torque:-

1. The Complex power output per phase

$$S_{1\phi} = VI_a^*$$

Taking  $E_f$  as the reference phasor

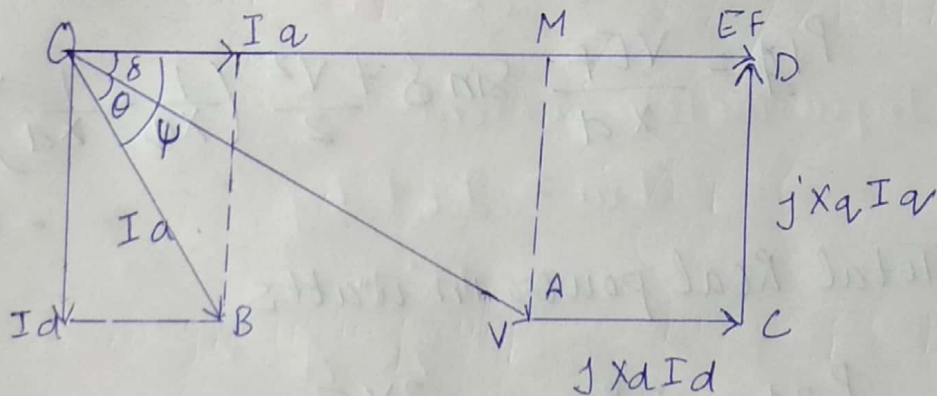
$$V = V \angle -\delta = V \cos \delta - jV \sin \delta$$

$$I_a = I_q - jI_d$$

$$I_a^* = I_q + jI_d$$

$$S_{1\phi} = VI_a^* = [V(\cos \delta - jV \sin \delta)(I_q + jI_d)]$$

2.



From the phasor diagram

$$X_q I_q = CD = AM = V \sin \delta$$

$$I_q = \frac{V \sin \delta}{X_q}$$

③

$$X_d I_d = AC = MD = OD - OM = E_f - V \cos \delta$$

$$I_d = \frac{E_f - V \cos \delta}{X_d}$$

(4) Substituting the value of  $I_q$  and  $I_d$

$$\begin{aligned} S_{1\phi} &= (V \cos \delta - jV \sin \delta) \left( \frac{V \sin \delta}{x_q} + j \frac{E_f - V \cos \delta}{x_d} \right) \\ &= \left[ \frac{VE_f}{x_d} \sin \delta + \frac{V^2}{2} \left( \frac{1}{x_q} - \frac{1}{x_d} \right) \sin 2\delta \right] \\ &\quad + j \left[ \frac{VE_f}{x_d} \cos \delta - \frac{V^2}{2x_d x_q} \{ (x_d + x_q) - (x_d - x_q) \cos 2\delta \} \right] \end{aligned} \quad \text{--- eqn (5)}$$

(5)  $S_{1\phi} = P_{1\phi} + jQ_{1\phi}$  --- eqn (6)

(6) the real power per phase in watts

$$P_{1\phi} = \frac{VE_f}{x_d} \sin \delta + \frac{V^2}{2} \left( \frac{1}{x_q} - \frac{1}{x_d} \right) \sin 2\delta \quad \text{--- eqn (7)}$$

(7) Total Real power in watts

$$P_{3\phi} = 3P_{1\phi} = \frac{3VE_f}{x_d} \sin \delta + \frac{3V^2}{2} \left( \frac{1}{x_q} - \frac{1}{x_d} \right) \sin 2\delta.$$

(8) The electromagnetic torque for a 3-phase synchronous machine is given by:

$$\tau_{em} = \frac{3P_{1\phi}}{\omega_m} = \frac{3}{2\pi N_s} \left( \frac{VE_f}{x_d} \sin \delta + \frac{x_d - x_q}{2x_d x_q} \sin 2\delta \right) \quad \text{--- eqn (8)}$$



(9)

The resulting torque has two components. The first term in eqn (8) represents the torque  $\pi_{exc}$  due to field excitation

$$\pi_{exc} = \frac{3VE_f}{2\pi n_s X_d} \sin \delta \quad \text{eqn (9)}$$

(10)

The second term in eqn (8) is known as reluctance torque,  $\pi_{rel}$ .

$$\pi_{rel} = \frac{3}{2\pi n_s} \left( \frac{X_d - X_q}{2X_d X_q} \right) \sin 2\delta$$