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$40 \times \frac{4}{16} + 240 \times \frac{1}{16}$

Set-5

Q1 Find the possible general solution of two dimensional laplace eqn using method of separation of variables.

Soln

laplace eqn in two dimension is given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{--- (1)}$$

let $u = XY$ --- (2)

$$\frac{\partial^2 u}{\partial x^2} = X''Y$$

$$\frac{\partial^2 u}{\partial y^2} = XY''$$

Substituting in eq (1)

$$X''Y + XY'' = 0$$

$$\frac{X''}{X} = -\frac{Y''}{Y} = k \text{ (let)} \quad \text{--- (3)}$$

$$\frac{\partial^2 X}{\partial x^2} - kX = 0 \quad \text{--- (4)}$$

$$\frac{\partial^2 Y}{\partial y^2} + kY = 0$$

solving eq (4) we get.

(i) When k is positive & $k = p^2$

$$X = C_1 e^{px} + C_2 e^{-px}$$

$$Y = C_3 \cos py + C_4 \sin py$$

(ii) When k is negative & $k = -p^2$

$$X = C_1 \cos px + C_2 \sin px$$

$$Y = C_3 e^{py} + C_4 e^{-py}.$$

$) < x^2$

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iii) when $k=0$

$$X = c_1 x + c_2,$$

$$Y = c_3 y + c_4$$

Thus, the various possible sol^n of laplace eq^n (2) are

$$u = (c_1 e^{px} + c_1 e^{-px})(c_3 \cos py + c_4 \sin py) \quad (5)$$

$$u = (c_1 \cos px + c_2 \sin px)(c_3 e^{py} + c_4 e^{-py}) \quad (6)$$

$$u = (c_1 x + c_2)(c_3 y + c_4) \quad (7)$$

From these three solutions, we have to choose that sol^n which is consistent with the Physical of the problem & the given boundary conditions.