Section-4

Q1) find the mean and variance of poisson disturbution. The disturbution, the number of ~~road accident~~ of road accident Per day in a City is poisson with mean 4. find the number of days when there will be,

1. No accident
2. At least 2 accident
3. At ~~m~~ most three accident
4. Between 2 and 5 ~~a~~ accident.

for the Poisson distribution, $P(r) = \frac{e^{-\lambda}\lambda^r}{r!}$

Mean of Poisson distribution:

$$\text{Mean}, \mu = \sum_{r=0}^{\infty} rP(r) = \sum_{r=0}^{\infty} r.\frac{e^{-\lambda}\lambda^r}{r!}$$

$$= e^{-\lambda}\sum_{r=0}^{\infty}\frac{r\lambda^r}{r!} = e^{-\lambda}\left(0+\frac{\lambda}{1!}+\frac{2\lambda^2}{2!}+\dots\right)$$

$$= \lambda e^{-\lambda}\left(1+\frac{\lambda}{1!}+\frac{\lambda^2}{2!}+\dots\right] = \lambda e^{-\lambda}.e^{\lambda} = \lambda$$

Variance of Poisson distribution:

$$\text{Variance}, \sigma^2 = \sum_{r=0}^{n} r^2P(r) - \mu^2$$

Notes

Call

$$= \sum_{r=0}^{\infty} r^2.\frac{\lambda^r e^{-\lambda}}{r!} - \lambda^2 = e^{-\lambda}\sum_{r=0}^{\infty}\frac{r^2\lambda^r}{r!} - \lambda^2$$

$$= e^{-\lambda}\left[\frac{1^2\lambda}{1!}+\frac{2^2\lambda^2}{2!}+\frac{3^2\lambda^3}{3!}+\frac{4^2\lambda^4}{4!}+\dots\right]-\lambda^2$$

$$= \lambda e^{-\lambda}\left[1+\frac{(1+1)\lambda}{1!}+\frac{(1+2)\lambda^2}{2!}+\frac{(1+3)\lambda^3}{3!}+\dots\right]-\lambda^2$$

$$\lambda e^{-\lambda}\left[\left(1+\frac{\lambda}{1!}+\frac{\lambda^2}{2!}+\frac{\lambda^3}{3!}+\dots\right)+\left(\frac{\lambda}{1!}+\frac{\lambda^2}{1!}+\frac{\lambda^3}{2!}+\dots\right)\right]-\lambda^2$$

$$= \lambda e^{-\lambda}\left[e^{\lambda}+\lambda\left(1+\frac{\lambda}{1!}+\frac{\lambda^2}{2!}+\dots\right)\right]-\lambda^2$$

$$= \lambda e^{-\lambda}\left[e^{\lambda}+\lambda e^{\lambda}\right]-\lambda^2$$

$$= \lambda e^{-\lambda}.e^{\lambda}(1+\lambda)-\lambda^2$$

$$= \lambda(1+\lambda)-\lambda^2 = \lambda$$

Hence The varriance of The Poisson disturbution is also λ

Notes

Call

Mean $r = 4$ Number of days, $N = 100$

(i) $P(r=0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-4} = 0.01831$

∴ Required number of days $= N \cdot P(r=0)$
$= 100 \times 0.01831 = 1.831 = 2$

(ii) $P(r \geq 2) = 1 - P(r<2) = 1 - [P(r=0) + P(r=1)]$

$= 1 - \left[e^{-4} + \frac{e^{-4}(4)}{1!}\right] = 1 - 5e^{-4} = 0.90842$

∴ Required number of days $= N \cdot P(r \geq 2)$
$= 100 \times 0.90842 = 90.842 = 91$.

(iii) $P(r \leq 3) = P(r=0) + P(r=1) + P(r=2) + P(r=3)$
$= \frac{e^{-4}(4)^0}{0!} + \frac{e^{-4}(4)^1}{1!} + \frac{e^{-4}(4)^2}{2!} + \frac{e^{-4}(4)^3}{3!}$

$= e^{-4} + 4e^{-4} + 8e^{-4} + \frac{64}{6}e^{-4} = 0.43347$

∴ Required number of days $= N \cdot P(r \leq 3)$
$100 \times 0.43347 = 43.347 = 43$

(iv) $P(2 < r < 5) = P(r=3) + P(r=4) = \frac{e^{-4}(4)^3}{3!} + \frac{e^{-4}(4)^4}{4!}$

Notes

Call

$= \left(\frac{64}{6} + \frac{256}{24}\right) e^{-4} = 0.3907$

∴ Required number of days
$= N \cdot P(2 < r < 5) = 100 \times 0.3907 = 39.07 = 39$.

Notes

Call