Section 2

Q1) Solve $x^2 r - y^2 t + px - qy = \log x$

let $x = e^X$, $y = e^Y$ so that $X = \log x$ and $Y = \log y$
and let $D = \frac{\partial}{\partial X} = D' \frac{\partial}{\partial Y}$ the the given equation reduce to

$$[D(D-1) - D'(D'-1) + D - D']z = X$$
$$(D^2 - D'^2)z = X$$

which is homogenous linear partial differential equal with coefficents.

$$CF = \phi(Y+X) + \phi_2(Y-X)$$

$$PI = \frac{1}{D^2 - D'^2}(X) = \frac{1}{(1)^2-(1)^2} \iint u \, du \, du$$

$$X = u = \int \frac{u^2}{2} du = \frac{u^3}{6} = \frac{X^3}{6}$$

$$z = \phi(Y+X) + \phi(Y-X) + \frac{X^3}{6}$$

$$= \phi(\log y + \log x) + \phi_2(\log y - \log x) + \frac{(\log x)^2}{6}$$

$$= \phi(\log xy) + \phi\left(\log \frac{y}{x}\right) + \frac{1}{6}(\log x)^3$$

Notes

Call

$z = f_1(xy) + f_2\left(\frac{y}{x}\right) + \frac{1}{6}(\log x)^3$