Q2) Solve: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ Subject to boundary conditions $u(0,y) = 0 = u(x,y)$, and $u(x,0) = u_0$, $\lim_{y\to\infty} u(x,y) = 0, 0<x<\pi$.

Given $\frac{\partial^2}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

$$u = XY$$

Let $Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} = 0$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k^2 \text{ (say)}$$

$$X = c_1 \cos kx + c_2 \sin kx, \quad Y = c_3 e^{ky} + c e^{-ky}$$

$$u = (c_1 \cos kx + c_2 \sin kx)(c_3 e^{ky} + c_4 e^{-ky})$$

$$u(0,y) = 0$$

$$0 = c_1(c_3 e^{ky} + c_4 e^{-ky})$$

$$c_1 = 0$$

from eq. (2.20.1),

$$u = \sin kx (A_1 e^{kx} + B_a e^{-b})$$

$$u(\pi, y) = 0$$

Notes

Call

$\sin K\pi = 0 \Rightarrow K = n$

$u = 0 \Rightarrow K = n$

$u = \sin \pi x \left( A_c e^{x} + B_c e^{xy} \right)$

$\lim u(x,y) = 0$, It is satisfies only when $A_a = 0$

from eq. (2.20.3),

$u = \sum B_a e^{-ay} \sin nx$