Section – 1

Q4

Ans ⇒ 4 $x^2\frac{d^2z}{dx^2} - 4xy\frac{d^2z}{dxdy} + 4y^2\frac{d^2z}{dy^2} + 6y\frac{dz}{dy} = x^3y^4$

Put $x = e^X$, $y = e^Y$ so that $X = \log x$ and $Y = \log y$ and let $D \equiv \frac{d}{dX}$, $D' \equiv \frac{d}{dY}$ and $DD' = \frac{d^2}{dXdY}$

then the given equation reduces to

$[D(D-1) - 4DD' + 4D'(D'-1) + 6D']z = e^{3X+4Y}$

$\Rightarrow [(D^2 - 4DD' + 4D'^2) - (D - 2D')]z = e^{3X+4Y}$

$\Rightarrow (D - 2D')(D - 2D' - 1)z = e^{3X+4Y}$

$CF = f_1(Y + 2X) + e^X f_2(y + 2X)$

$f_1(\log y + 2\log x) + x f_2(\log y + 2\log x)$

$f_1(\log yx^2) + x f_2(\log yx^2) = g_1(yx^2)) + xy_2(yx^2)$

$PI = \frac{1}{D\,2D'-1}\left[\frac{1}{D-2D'}e^{3X+4Y}\right]$

$= \frac{1}{D-2D'-1}\left[\frac{1}{3-0}\int e^U dU\right]$ where $3X+4Y = U$

$\frac{1}{D-2D'-1}\left[\frac{1}{5}e^{3X+4Y}\right]$

$= -\frac{1}{5}\left[\frac{1}{D-2D'-1}e^{3X+4Y}\right]$

$= -\frac{1}{5}\left[\frac{1}{3-8-1}e^{3X+4Y}\right] = \frac{1}{30}e^{3X+4Y}$

$= \frac{1}{30}x^3y^4$

Hence the complete solution is

$z = CF + PI = g_1(yx^2) + xy_2(yx^2) + \frac{1}{30}x^3y^4$

where g_1 & g_2 are arbitrary functions.