

Q2) Prove that the language $L = \{0^n 1^n \mid n \text{ is perfect cube}\}$ is not regular. Construct the CFG for the regular expression $(0+1)^*$

Solⁿ

1. Let us assume L is regular, let p be a constant provided by the pumping lemma.
2. Let w be the string 0^3 . This string is in L , and is of length at least p . So w can be written as xyz with $|xy| \leq p$ and $y \neq \epsilon$.
3. Pumping lemma states that if $xyz \in L$ then xy^kz also is in L for any $k \geq 1$. Since y contains at least one 0 in y since y is not ϵ .
4. Let us assume for $k=2$, the resulting string is $xyyz$.
5. So it suffices to show that $p^3 \neq p^3 + p < (p+1)^3$.
The first inequality holds since $p > 0$.

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6. The claim is established, so $xyyz$ is not in L .

7. This contradicts the pumping lemma, so our original assumption, that L was regular is not incorrect.

~~CFG for~~ CFG for $(0+1)^*$

The CFG can be given by,
Production rule

(P):

$S \rightarrow 0S \mid 1S$

$S \rightarrow \epsilon$

The rules are in the combination of 0's & 1's with the start symbol. Since $(0+1)^*$ indicates $\{\epsilon, 0, 1, 01, 10, 00, 11, \dots\}$. In this set, ϵ is a string, so in the rule, we can set the rule $S \rightarrow \epsilon$.