SEC - 1

(3) one dimensional wave eq^n is given by

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{where } c^2 = \frac{T}{m} \quad \text{--- (1)}$$

T = Tension in the string.
m = Mass per unit length of the string.

Soln. of one dimensional wave eq^n is done by the method of seperation of variable.

Let

$$u = X(x)\,T(t) \quad \text{--- (2)}$$

where X is a function of x only and T is a function of t only

Differentating eq^n (11) partially w.r.t. x and t respectivally and putting the values in eq^n (1)

$$\frac{\partial^2 u}{\partial t^2} = X \frac{\partial^2 T}{\partial t^2} \quad \text{and}$$

$$\frac{\partial^2 u}{\partial x^2} = T \frac{\partial^2 X}{\partial x^2}$$

from one dimentional wave length eq^n

$$X \frac{\partial^2 T}{\partial T^2} = c^2 T \frac{\partial^2 X}{\partial x^2}$$

Seperating the value,

Case-1 $\frac{1}{X}\frac{\partial^2 T}{\partial t^2} = \frac{1}{C^2 T}\frac{\partial^2 X}{\partial x^2} = -K^2$

and $\frac{1}{C^2 T}\frac{\partial^2 T}{\partial t^2} = -K^2$

or $\frac{\partial^2 X}{\partial x^2} + K^2 X = 0$ and $\frac{\partial^2 T}{\partial t^2} + K^2 C^2 T = 0$

or $(D^2 + K^2)X = 0$ and $(D^2 + K^2 C^2)T = 0$

Auxiliary eqn are $m^2 + K^2 = 0$
and $m^2 + K^2 C^2 = 0$
$m = \pm Ki$ and $m = \pm KCi$

Thus complementry functions are
$X = C_1 \cos Kx + C_2 \sin Kx$
$T = C_3 \cos KCt + C_4 \sin KCT$
$u = XT$
$\Rightarrow u = (C_1 \cos Kx + C_2 \sin Kx)(C_3 \cos KCt + C_4 \sin KCt)$

Case II

when $\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = k^2$ and

$$\frac{1}{c^2 T}\frac{\partial^2 T}{\partial t^2} = k^2$$

$m = k^2$ and $m^2 = k^2 c^2$

$m = \pm k$ and $m \pm kc$

$\Rightarrow X = c_5 e^{kx} + c_6 e^{-kct}$

Thus $u = (c_5 e^{kx} + c_6 e^{-kx})$
$(c_7 e^{kct} + c_8 e^{-kct})$ —— (4)

Case III

when $\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = 0$ and $\frac{1}{c^2 T}$

$$\frac{\partial^2 T}{\partial t^2} = 0$$

$\Rightarrow X = c_9 + c_{10}x$ and $T = c_{11} + c_{12}t$

Thus $u = (c_9 + c_{10}x)(c_{11} + c_{12}t)$

—— (5)