SEC-5

Q2> Sin R and G are negligible, transmission line equations becomes

$$\frac{\partial e}{\partial x} = -L \frac{\partial i}{\partial t} \quad \text{---- (i)}$$

and
$$\frac{\partial i}{\partial x} = -C \frac{\partial e}{\partial t} \quad \text{---- (ii)}$$

For elimination of i, differentiating eq. (i) partially w.r.t x and eq. (ii) partially w.r.t t, we have

$$\frac{\partial^2 e}{\partial x^2} = -L \frac{\partial^2 i}{\partial x \partial t} \text{ and } \frac{\partial^2 i}{\partial t \partial x} = C \frac{\partial^2 e}{\partial t^2}$$

Hence,
$$\frac{\partial^2 e}{\partial x^2} = LC \frac{\partial^2 e}{\partial t^2} \quad \text{---- (iii)}$$

The initial conditions are $i(x,0) = i_0$, $e(x,0) = e_0 \sin \frac{\pi x}{l}$ ---- (iv)

Since the ends are suddenly grounded, the boundary conditions are

$e(0,t) = e(l,t) = 0$ ---- (V)

Also $i = i_0$ (constant) when $t = 0$

$\therefore \frac{\partial i}{\partial x} = 0$ which gives $\frac{\partial e}{\partial t} = 0$ when $t = 0$ --(vi)

Now let $e = XT$ be a solution of eq (iii) where X is a function of x only and T is a function of t only.

$$\frac{\partial^2 e}{\partial x^2} = XT \text{ and } \frac{\partial^2 e}{\partial t^2} = XT''$$

$\therefore$ From eq. (iii) $X''T = LCXT''$

Separating the variables $\frac{X''}{X} = LC \frac{T''}{T} = -p^2$ (say)

This leads to the ordinary differential equations

$$\frac{d^2X}{dx^2} + p^2 x = 0 \text{ and } \frac{d^2T}{dt^2} + \frac{p^2}{LC} T = 0$$

$$X = C_1 \cos px + C_2 \sin px$$
$$T = C_3 \cos \frac{pt}{\sqrt{LC}} + C_4 \sin \frac{pt}{\sqrt{LC}}$$

$$e = XT = (C_1 \cos px + C_2 \sin px)\left(C_3 \cos \frac{pt}{\sqrt{LC}} + C_4 \sin \frac{pt}{\sqrt{LC}}\right) \quad \text{---- (vii)}$$

Applying the boundary conditions eq (V) in eq. (VII), we get

$C_1 = 0$ and $p = \frac{n\pi}{l}$, n being an integer

$\therefore$ Eq. (vii) becomes

$$e = C_2 \sin \frac{n\pi x}{l}\left(C_3 \cos \frac{n\pi t}{l\sqrt{LC}} + C_4 \sin \frac{n\pi t}{l\sqrt{LC}}\right)$$

or
$$e = \sin \frac{n\pi x}{l}\left(A \cos \frac{n\pi t}{l\sqrt{LC}} + B \sin \frac{n\pi t}{l\sqrt{LC}}\right)$$

where $A = C_2C_3$ and $B = C_2C_4$ ---- (viii)

$$\frac{\partial e}{\partial t} = \sin \frac{n\pi x}{l}\left(-\frac{An\pi}{l\sqrt{LC}} \sin \frac{n\pi t}{l\sqrt{LC}} + \frac{Bn\pi}{l\sqrt{LC}} \cos \frac{n\pi t}{l\sqrt{LC}}\right)$$

Since $\frac{\partial e}{\partial t} = 0$ when $t = 0$, we get $B = 0$

$\therefore$ From eq. (viii) $e = A \sin \frac{n\pi x}{l} \cos \frac{n\pi x}{l\sqrt{LC}}$

By superposition, $e = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{l} \cos \frac{n\pi x}{l\sqrt{LC}}$ is also a solution.

But $e = e_0 \sin \frac{\pi x}{l}$ when $t = 0$

$\therefore$ $e_0 \sin \frac{\pi}{l} = \sum_{n-1}^{\infty} A_n \sin \frac{n\pi x}{l}$

$$A_1 = e_0 \text{ and } A_2 = A_3 = \cdots = 0$$

$\Rightarrow$

Hence, $e = e_0 \sin \frac{\pi x}{l} \cos \frac{\pi t}{l\sqrt{LC}}$

$$-L \frac{\partial i}{\partial t} = \frac{\partial e}{\partial x}$$

Now, $\frac{\partial i}{\partial t} = -\frac{1}{L} \cdot \frac{e_0 \pi}{l} \cos \frac{\pi x}{l} \cos \frac{\pi t}{l\sqrt{LC}}$

Integrating w.r.t. t, regarding x as constant

$$i = -\frac{e_0 \pi}{Ll} \cos \frac{\pi x}{l} \cdot \frac{l\sqrt{LC}}{\pi} \sin \frac{\pi t}{l\sqrt{LC}} + f(x)$$

where $f(x)$ is an arbitrary contant function

since $i = i_0$ when $t = 0$, we have $i_0 = 0 + f(x)$

or $f(x) = i_0$

$\therefore$ From eq (X) we have

$$i = i_0 - e_0 \sqrt{\frac{C}{L}} \cos \frac{\pi x}{l} \sin \frac{\pi t}{l\sqrt{LC}}$$