

SEC-2

(2) Given $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Let $u = xy$

$$y \frac{\partial^2 x}{\partial x^2} + x \frac{\partial^2 y}{\partial y^2} = 0$$

$$\frac{1}{x} \frac{\partial^2 x}{\partial x^2} = -\frac{1}{y} \frac{\partial^2 y}{\partial y^2} = k^2 \text{ (say)}$$

$$X = C_1 \cos kx + C_2 \sin kx, Y$$

$$= C_3 e^{ky} + C_4 e^{-ky}$$

$$u = (C_1 \cos kx + C_2 \sin ky)$$

$$(C_3 e^{ky} + C_4 e^{-ky}) \quad \text{--- (1)}$$

$$u(0, y) = 0$$

$$0 = C_1 (C_3 e^{ky} + C_4 e^{-ky})$$

$$C_1 = 0$$

from eqn (1)

$$u = \sin kx (A_n e^{ky} + B_n e^{-ky}) \quad \text{--- (2)}$$

$$u(\pi, y) = 0$$

$$\sin k\pi = 0 \Rightarrow k = n$$

$$u = \sin nx (A_n e^{ny} + B_n e^{-ny}) \quad \text{--- (3)}$$

Lim $u(x, y) = 0$, it satisfy

only, when $A_n = 0$

from eqn. (3)

$$u = \sum B_n e^{-ny} \sin nx \quad \text{--- (4)}$$

$$\text{Now } u(x, 0) = u_0$$

$$u_0 = \sum B_n \sin nx$$

$$B_n = \frac{2}{\pi} \int_0^\pi u_0 \sin nx \, dx = \frac{2u_0}{\pi}$$

$$\left[\frac{\cos nx}{n} \right]_0^\pi = \frac{-2u_0}{\pi} \left[\frac{(-1)^n - 1}{n} \right]$$

from eqn (4)

$$u = \sum \frac{-2u_0}{\pi n} \left[(-1)^n - 1 \right] e^{-ny} \sin nx$$