SEC-1

(4) Put $x = e^{X}$, $y = e^{Y}$ so that $X = \log x$ and $Y = \log y$ and let $D \equiv \frac{\partial}{\partial X}$, $D' \equiv \frac{\partial}{\partial Y}$ and $DD' = \frac{\partial^2}{\partial X \partial Y}$ then the given eq^n reduced to

$$[D(D-1) - 4DD' + 4D'(D'-1) + 6D']z$$

$$= e^{3X+4Y}$$

$$\Rightarrow [D^2 - 4DD' + 4D'^2) - (D-2D')]z$$

$$= e^{3X+4Y}$$

$$\Rightarrow (D-2D')(D-2D'-1)z = e^{3X+4Y}$$

its $CF = f_1(Y+2X) + e^{X}f_2(Y+2X)$

$= f_1(\log y + 2\log x) + x f_2(\log y + 2\log x)$

$= F_1(\log y . x^2) + x f_2(\log y . x^2) = g_1(yx^2))$

$+ x g_2 (yx)^2$

$$PI = \frac{1}{D \cdot 2D' - 1}\left[\frac{1}{D-2D'} e^{3X+4Y}\right]$$

$$= \frac{1}{D-2D'-1}\left[\frac{1}{3-8}\int e^u\,du\right] \quad \text{where } 3X+4Y=u$$

$$= \frac{1}{D-2D'-1}\left[-\frac{1}{5}e^{3X+4Y}\right]$$

$$= -\frac{1}{5}\left[\frac{1}{D-2D'-1}e^{3X+4Y}\right]$$

$$= -\frac{1}{5}\left[\frac{1}{3-8-1}e^{3X+4Y}\right]$$

$$= \frac{1}{30}e^{3X+4Y} = \frac{1}{30}x^3y^4$$

Hence the complete solution is

$$z = CF + PI = g_1(y.x^2) + xg_1(yx^2) + \frac{1}{30}x^3y^4$$

where g_1 and g_1 are abitrary function.