# SEC - 2

① Let $x = e^X$, $y = e^Y$ so that $X = \log x$ and $Y = \log y$

and Let $D = \frac{\partial}{\partial X}$

and $D' \equiv \frac{\partial}{\partial Y}$ then the given eq^n reduce to

$$[D(D-1) - D'(D'-1) + D - D']z = X \quad \text{——— ①}$$

$$\Rightarrow (D^2 - D'^2)z = X$$

which is a homogeneous partial differential eq^n, with constant coefficient

$\therefore CF = \phi_1(Y+X) + \phi_2(Y-X)$

and

$$PI = \frac{1}{D^2 - D'^2}(X) = \frac{1}{(1)^2 - (0)^2}$$

$$\iint u\,du\,du$$

where $X = u$

$$= \int \frac{u^2}{2}\,du = \frac{u^3}{6} = \frac{X^3}{6}$$

Hence soln to eq^n ① is

$$z = \phi_1(Y+X) + \phi_2(Y-X) + \frac{X^3}{6}$$

$$= \phi_1(\log y + \log x) + \phi_2(\log y - \log x) + (\log x)^3/6$$

Therefore the complete
Soln. to the given differential
eqn is

$z = \phi_1(\log xy) + \phi_2(\log y/x)$

$+ \frac{1}{6}(\log x)^3$

$= z\, f_1(xy) + F_2(y/x) + \frac{1}{6}(\log x)^3$

where $F_1$ and $F_2$ are
arbitrary function.