SEC - 1

(2) Let the eq^n of temperature distribution is

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{---} \quad (1)$$

$$u = X(x)\, T(t)$$

$$X \frac{\partial T}{\partial t} = c^2 T \frac{\partial^2 X}{\partial x^2}$$

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = \frac{1}{C^2 T} \frac{\partial T}{\partial T} = -K^2 \text{ (let)}$$

$$X = C_1 \cos Kx + C_2 \sin Kx$$

$$T = C_3 e^{-K^2 C^2 t}$$

Thus $u = (A_n \cos Kx + B_n \sin Kx)\, e^{-K^2 C^2 t}$ --- (2)

Given boundary and initial condition are

$$\left(\frac{\partial u}{\partial x}\right)_{x=0} = 0$$

$$\left(\frac{\partial u}{\partial x}\right)_{x=a} = 0$$

$$u(x, 0) = x(a - x), \quad 0 < x < a$$

$$\frac{\partial u}{\partial x} = (-K A_n \sin Kx + B_n K \cos Kx)\, e^{-K^2 C^2 t}$$

$0 = B_n k e^{-k^2 c^2 t}$

$B_n = 0$

from eq^n (11)

$u = A_n \cos kx \, e^{-k^2 c^2 t}$ — (3)

$\frac{\partial u}{\partial x} = k A_n \sin k \, e^{-k^2 c^2 t}$

$0 = k A_n \sin k a \, e^{-k^2 c^2 t}$

$k = \frac{n\pi}{a}$

$$u = \sum_{n=0}^{\infty} A_n \cos\left(\frac{n\pi x}{a}\right) e^{-n^2\pi^2 c^2 t/a^2} \quad \text{— (4)}$$

Now at $t = 0$

$$u(x,0) = \sum A_n \cos\left(\frac{n\pi x}{a}\right)$$

$$A_n = \frac{2}{a}\int_0^a u(x,0)\cos\left(\frac{n\pi x}{a}\right)dx$$

$$A_n = \frac{2}{a}\int_0^a x(a-x)\cos\left(\frac{n\pi x}{a}\right)dx$$

$$= \frac{2}{a}\left[(ax-x^2)\left(\frac{a}{n\pi}\sin\frac{n\pi x}{a}\right) - (a-2x)\left(\frac{-a^2}{n^2\pi^2}\cos\frac{n\pi x}{a}\right) + (-2)\left(\frac{-a^3}{n^3\pi^3}\sin\frac{n\pi x}{a}\right)\right]_0^a$$

$$A_n = \frac{2}{a}\left[-(\cancel{a}\,a)\left(\frac{-a^2}{n^2\pi^2}\cos n\pi\right) + a\left(\frac{-a^2}{n^2\pi^2}\right)\right]$$

$$= \frac{2a^2}{n^2\pi^2}\left[1+\cos n\pi\right]$$

Thus $(x,t) = \sum_0^{\infty} \frac{-2a^2}{n^2\pi^2}(1+\cos nx \cos\left(\frac{n\pi x}{a}\right)) e^{-n^2\pi^2 c^2 t/a^2}$