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a) Here Larrange's subsidiary eqn are.

$$\frac{dx}{x^2-yz} = \frac{dy}{y^2-zx} = \frac{dz}{z^2-xy}$$

$$\therefore \frac{dx-dy}{(x-y)(x+y+z)} = \frac{dy-dz}{(y-z)(x+y+z)}$$

$$\therefore \frac{dz-dx}{(z-x)(x+y+z)}$$

Taking the first two members, we have.

$$\frac{dx-dy}{x-y} = \frac{dy-dz}{y-z}$$

which on integration gives

$$\log(x-y) = \log(y-z) + \log a$$

$$\text{or } \log\left(\frac{x-y}{y-z}\right) = \log a$$

$$\text{or } \frac{x-y}{y-z} = a \quad \text{--- (1)}$$

Similarly, taking the last two members, we obtain

$$\frac{y-z}{z-x} = b \quad \text{--- (2)}$$

from eqn (1) & (2) the general solution is

$$\phi\left(\frac{x-y}{y-z}, \frac{y-z}{z-x}\right) = 0$$

b) $x^2z^p + x^2z^p = y^2x$

The subsidiary eqn are.

$$\frac{dx}{y^2z} = \frac{dy}{x^2z} = \frac{dz}{y^2x}$$

The first two fraction give $x^2 dx = y^2 dy$

Integrating, we get $x^3 - y^3 = a$ --- (1)

Again, the first and third fraction give $x dx = z dz$

Integrating, we get $x^2 - z^2 = b$ --- (2)

Hence, from eqn (1) & (2) the soln is $x^3 - y^3 = f(x^2 - z^2)$