SEC - 5

Q1) Laplace equation in two dimension is given by —

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{---} (1)$$

Let

$$u = XY \quad \text{---} (2)$$

Then

$$\frac{\partial^2 u}{\partial x^2} = X''Y$$

$$\frac{\partial^2 u}{\partial y^2} = XY''$$

substituting in eq. (1)

$$X''Y + XY'' = 0 \quad \text{---} (3)$$

or

$$\frac{X''}{X} = -\frac{Y''}{Y} = k \text{ (say)}$$

$$\frac{d^2 X}{dx^2} - kX = 0 \quad \text{---} (4)$$

Now.

$$\frac{d^2 Y}{dy^2} + kY = 0$$

Solving eq. (4), we get

(i) When k is positive and $k = p^2$

$$X = c_1 e^{px} + c_2 e^{px}$$

$$Y = c_3 \cos py + c_4 \sin py$$

(ii) when k is negative and $k = -p^2$

$$X = c_1 \cos px + c_2 \sin px, \quad Y = c_3 e^{py} + c_4 e^{-py}$$

(iii) $R = 0$

$$X = c_1 x + c_2, \; Y = c_3 y + c_4$$

Thus, the various possible solutions of Laplace equation (2) are

$$u = (c_1 e^{px} + c_2 e^{-px})(c_3 \cos py + c_4 \sin py) \quad (v)$$

$$u = (c_1 \cos px + c_2 \sin px)(c_3 e^{py} + c_4 e^{-py}) \quad (v$$

$$u = (c_1 x + c_2)(c_3 y + c_4) \quad \text{------} \quad (vii)$$

From these three solutions, we have to choose that solution which is consistent with the physical nature of the problem and the given boundary conditions.