SEC-6

① Explain the test of significance of difference of means. Draw C-chart for the following data pertaining to the number of foreign colour threads (considered as defect) in 15 pieces of cloth of 2m × 2m in a certain make of synthetic fiber and state your conclusions.

7, 12, 13, 20, 21, 5, 4, 3, 10, 8, 0, 9, 6, 7, 20

i) Given two independent samples $x_1, x_2, x_3, \ldots, x_n$ and $y_1, y_2, y_3, \ldots, y_{n_2}$ with means $\bar{x}$ and $\bar{y}$ and standard deviations σ_x and σ_y from a normal population with the same variance, we have to test the hypothesis that the population means μ_1 and μ_2 are the same.

For this, we calculate

$$t = \frac{\bar{x} - \bar{y}}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad \text{— ①}$$

where

$$\bar{x} = \frac{1}{n_1}\sum_1^{n_1} x_1, \quad \bar{y} = \frac{1}{n_2}\sum_1^{n_2} y_1$$

and

$$\sigma^2 = \frac{1}{n_1+n_2-2}\left[(n_1-1)\sigma_x^2 + (n_2-1)\sigma_y^2\right] = \frac{1}{n_1+n_2-2}\left\{\sum_1^{n_1}(x_i-\bar{x})^2 + \sum_1^{n_2}(y_i-\bar{y})^2\right\}$$

It can be shown that the variate t defined by eqn ① follows the t-distribution with n_1+n_2-2 degree of freedom.

If the calculated value of $t > t_{0.05}$, the difference between the sample means is said to be significant at 5% level of significance.

If $t > t_{0.01}$, the difference is said to be significant at 1% level of significant.

If $t < t_{0.05}$ the data is said to be consistent with

with hypothesis that $\mu_1 = \mu_2$

ii> Here, we have number of cloth pieces $= 15$

a> The total number of defects (C)

$= 7+12+3+20+21+5+4+3+10+8+0+9+6+7+20 = 135$

b> The average number of defects ($\bar{C}$)

$= \frac{\text{Total no. of defects}}{\text{Total number of samples}}$

$= \frac{\sum C}{n} = \frac{\cancel{135}^{\,9}}{\cancel{15}} = 9$

iii> The 3-σ control limits for C-chart are given by

central limit line

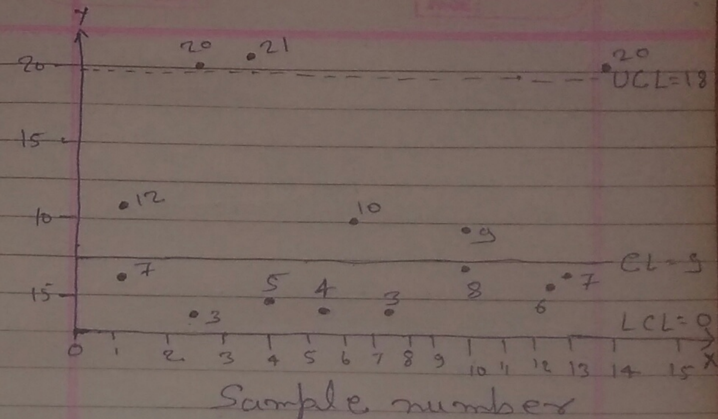
$= \bar{C} = 9$

$UCL = \bar{C} + 3\sqrt{\bar{C}} = (9+3\sqrt{9})$

$= 9+9 = 18$

$LCL = \bar{C} - 3\sqrt{\bar{C}} = 9 - 3\sqrt{9}$

$= 9-9 = 0$



Since three sample points are outside the limits, the process is not under Statical protocol.