SEC - 6

(2.) Prove that the language $L = \{0^n \mid n$ is perfect cube$\}$ is not regular. Construct the CFG for the regular expression $(0+1)^*$

Let us assume L is regular. Let p be a constant provided by the pumping lemma

Let w be the string 0^{p^3}. This string is in L and is of length at least p. So w can be written as xyz with $|xy| \le p$ and y not ε

Pumping lemma states that if $xyz \in L$ then xy^kz also in L. y contains at least one 0 in y since y is not ε.

Pumping lemma states that if $xyz \in L$ then Let us assume for $k = 2$ the resulting string is $xyyz$

we have to show that the length of $xyyz$ is not a perfect cube. Let q be the length of y. then of $xyyz$ is $p^3 + q$

So it suffices to show that $p^3 < p^3 + q < (p+1)^3$. The first inequality holds since $q > 0$ (since y was not ε)

For second inequality, we compute $(p+1)^3 = p^3 + 3p^2 + 3p + 1$ and this is greater than $p^3 + q$ since $q < p$ since $|xy| \le p$).

The claim is established. So $xyyz$ is not in L

This contradicts is the pumping lemma. So our original assumption that L was regular is not incorrect.