Sec→4

Q1) Find the mean & variance of poisson distribution.

The distribution of the number of road accidents per day in a city is poisson with mean 4.
find the number of days out of 100 day when there will be.

(i) no accident

(ii) at least 2 accident

(iii) at most 3 accident

(iv) between 2 & 5 accidents.

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Sol^n

For the poisson distribution, $P(r) = \frac{e^{-\lambda}\lambda^r}{r!}$

Mean of poisson distribution:

$$\text{mean } \mu = \sum_{r=0}^{\infty} r P(r) = \sum_{r=0}^{\infty} r \cdot \frac{e^{-\lambda}\lambda^r}{r!}$$

$$= e^{-\lambda}\sum_{r=0}^{\infty} \frac{r\lambda^r}{r!} = e^{-\lambda}\left(0 + \frac{\lambda}{1!} + \frac{2\lambda^2}{2!} + \cdots\right)$$

$$= \lambda e^{-\lambda}\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right)$$

$$= \lambda e^{-\lambda} \cdot e^{\lambda} = \lambda$$

Variance of poisson distribution.

$$\sigma^2 = \sum_{r=0}^{\infty} r^2 P(r) - \mu^2$$

$$= \sum_{r=0}^{\infty} r^2 \frac{\lambda^r e^{-\lambda}}{r!} - \lambda^2 = e^{-\lambda}\sum_{r=0}^{\infty} \frac{r^2\lambda^r}{r!} - \lambda^2$$

$$= e^{-\lambda}\left[\frac{1^2\lambda}{1!} + \frac{2^2\lambda^2}{2!} + \frac{3^2\lambda^3}{3!} + \frac{4^2\lambda^4}{4!} + \cdots\right] - \lambda^2$$

$$= \lambda e^{-\lambda}\left[1 + \frac{2\lambda^2}{1!} + \frac{3\lambda^2}{2!} + \frac{4\lambda^3}{3!} + \cdots\right] - \lambda^2$$

$$= \lambda e^{-\lambda}\left[1 + \frac{(1+1)\lambda}{1!} + \frac{(1+2)\lambda^2}{2!} + \frac{(1+3)\lambda^3}{3!} + \cdots\right] - \lambda^2$$

$$= \lambda e^{-\lambda}\left[\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots\right) + \left(\frac{\lambda}{1!} + \frac{\lambda^2}{1!} + \frac{\lambda^3}{2!}\right.\right.$$

$$= \lambda e^{-\lambda}\left[e^{\lambda} + \lambda\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right)\right] - \lambda^2$$

$$= \lambda e^{-\lambda}\left[e^{\lambda} + \lambda e^{\lambda}\right] - \lambda^2$$

$$= \lambda e^{-\lambda} \cdot e^{\lambda}(1+\lambda) - \lambda^2$$

$\therefore = \lambda(1+\lambda) - \lambda^2 = \lambda.$

Here, the variance of poisson distribution is also λ.

Mean $\lambda = 4$, Number of days, $N = 100$

(i) $P(r=0) = \dfrac{e^{-\lambda}\lambda^0}{0!} = e^{-4} = 0.01831$

$\therefore$ Required number of days $= N.P(r=0)$
$= 100 \times 0.01831 = 1.831 \simeq 2$

(ii) $P(r \geq 2) = 1 - P(r<2) = 1 - [P(r=0) + P(r=1)]$
$= 1 - \left[e^{-4} + \dfrac{e^{-4}(4)}{1!}\right] = 1 - 5e^{-4}$
$= 0.90842$

$\therefore$ Required number of days $= N.P(r \geq 2)$
$= 100 \times 0.90842 = 90.842 \simeq 91$

(iii) $P(r \leq 3) = P(r=0) + P(r=1) + P(r=2) + P(r=3)$
$= \dfrac{e^{-4}(4)^0}{0!} + \dfrac{e^{-4}(4)^1}{1!} + \dfrac{e^{-4}(4)^2}{2!} + \dfrac{e^{-4}(4)^3}{3!}$
$= e^{-4} + 4e^{-4} + 8e^{-4}\frac{64}{6}e^{-4}$
$= 0.43347$

$\therefore$ Required number of days
$= N.P(r \leq 3)$
$= 100 \times 0.43347 = 43.347 \simeq 43$

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(iv) $P(2 < r < 5) = P(r=3) + P(r=4) =$

$$\frac{e^{-4}(4)^3}{3!} + \frac{e^{-4}(4)^4}{4!}$$

$$= \left(\frac{64}{6} + \frac{256}{24}\right)e^{-4} = 0.3907$$

$\therefore$ required number of days

$= N \cdot P(2 < r < 5) = 100 \times 0.3907$

$= 39.07 \simeq 39.$