Sec → 3

Q(2) Find the four central moments & discuss skewness & kurtosis & also karl pearson skewness for the frequency distribution given below:

Range of Expendin ₹100/month	2-4	4-6	6-8	8-10	10-12
no. of families	38	292	389	212	69

Soln

Range	f	x	x-A	(x-A)f	f(x-A)²	f(x-A)³	f(x-A)⁴	C.F
2-4	38	3	-4	-152	608	-2432	9728	38
4-6	292	5	-2	-584	1168	-2336	4672	330
6-8	389	7(A)	0	0	0	0	0	719
8-10	212	9	2	424	848	1696	3392	931
10-12	69	11	4	276	1104	4416	17664	1000
	1000			-36	3728	1344	35456	

$$\mu_1' = \frac{\Sigma(x-A)f}{\Sigma f} = \frac{-36}{1000} = -0.036$$

$$\mu_2' = \frac{\Sigma(x-A)^2 f}{\Sigma f} = \frac{3728}{1000} = 3.728$$

$$\mu_3' = \frac{\Sigma(x-A)^3 f}{\Sigma f} = \frac{1344}{1000} = 1.344$$

$$\mu_4' = \frac{\Sigma(x-A)^4 f}{\Sigma f} = \frac{35456}{1000} = 35.456$$

Central moments

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - (\mu_1')^2 = 3.728 - (-0.036)^2$$

$$\mu_2 = 3.726704$$

$$\mu_3 = \mu_3'' - 3\mu_2''\mu_1'' + 2(\mu_1'')^3$$

$$= 1.344 - 3(3.728)(-0.036) + 2(-0.036)^3$$

$$\boxed{\mu_3 = 1.74653}$$

$$\mu_4 = \mu_4'' - 4\mu_1''\mu_3'' + 6\mu_2''(\mu_1'')^2 - 3(\mu_1'')^4$$

$$= 35.456 - 4\times(-0.036)(1.344) + 6(3.728)(-0.036)^2 - 3(-0.036)^4$$

$$\mu_4 = 35.6785.$$

Skewness

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(1.74653)^2}{(3.726704)^3}$$

$$= 0.0589$$

$$\beta_1 > 0$$

Positively skewed

kurtosis

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{35.6785}{(3.726704)^2} = 2.5689 < 3$$

Platykurtic

Karl pearson's coefficient of skewness $= \frac{3(\text{mean} - \text{median})}{SD}$

$$\text{mean} = A + \frac{\Sigma(x+A)f}{\Sigma f} = 7 + \frac{(-36)}{1000} = 6.964$$

$$\text{median} = l + \frac{\frac{N}{2} - C.f}{f} \cdot i = 6 + \frac{\frac{1000}{2} - 330}{389} \times 2$$

$$= 6 + \frac{170}{389} \times 2$$

$= 6.874$

$$SD = \sqrt{\frac{\Sigma f(x-A)^2}{\Sigma f} - \left(\frac{\Sigma f(x-A)}{\Sigma f}\right)^2}$$

$$= \sqrt{\frac{3728}{1000} - \left(\frac{-36}{1000}\right)^2}$$

$$= \sqrt{3.728 - 0.001296}$$

$$= \sqrt{3.726} = 1.930$$

Karl pearson's coefficient of skewness

$$= \frac{3(\text{mean} - \text{median})}{SD}$$

$$= \frac{3(6.964 - 6.874)}{1.930}$$

$S_k = 0.1398$

$9 < 3$

$S_k > 0$

Distribution is (+)vely skewed.