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(Q1) Is the function defined as follows a density function?

$$f(x) = e^{-x}, \quad x \geq 0$$
$$= 0, \quad x < 0$$

If so, determine the probability that the variate having this density will fall in the interval $(1, 2)$? Also find the cumulative probability function $F(2)$?

Soln

$f(x) \geq 0$ for every x in $(1, 2)$ &

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty} e^{-x}\,dx = 1$$

Hence the function $f(x)$ satisfies the requirements for a density functions.

Required probability $= P(1 \leq x \leq 2)$

$$= \int_{1}^{2} e^{-x}\,dx = e^{-1} - e^{-2}$$
$$= 0.368 - 0.135 = 0.233$$

This probability is equal to the shaded area in fig.

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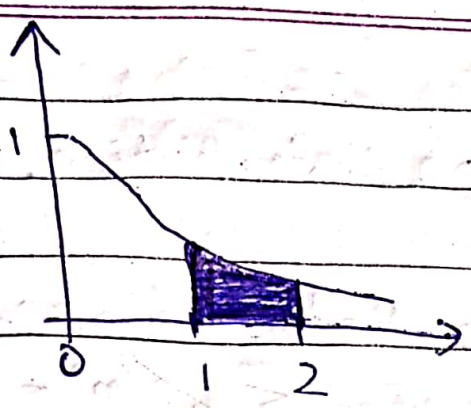

Cumulative probability function P(z):

$$\int_{-\infty}^{2} f(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{2} e^{-x}\,dx = 1 - e^{-2}$$

$$= 1 - 0.135 = 0.865$$

which shown in fig.