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$\frac{n\pi ct}{l}$ — (7)

(Q4) Solve the linear partial diff. eqn.

$$x^2\frac{\partial^2 z}{\partial x^2} - 4xy\frac{\partial^2 z}{\partial x \partial y} + 4y^2\frac{\partial^2 z}{\partial y^2} + 6y\frac{\partial z}{\partial y} = x^3y^4$$

Solⁿ

Put $x = e^X$, $y = e^Y$ so that $X = \log x$ & $Y = \log y$

& let $D = \frac{\partial}{\partial X}$, $D' = \frac{\partial}{\partial Y}$ &

— (9)

$DD' = \frac{\partial^2}{\partial X \partial Y}$ then the given eqn reduces to

$$\Rightarrow [D(D-1) - 4DD' + 4D'(D'-1) + 6D']z = e^{3X+4Y}$$

$$\Rightarrow [(D^2 - 4DD' + 4D'^2) - (D - 2D')]z = e^{3X+4Y}$$

$$\Rightarrow (D-2D')(D-2D'-1)z = e^{3X+4Y}$$

its,

$$CF = f_1(Y-2X) + e^X f_2(Y+2X)$$

$$= f_1(\log y + 2\log x) + x f_2(\log y + 2\log x)$$

$$= f_1(\log yx^2) + x f_2(\log yx^2) = g_1(yx^2) + x g_2(yx)^2$$

$$P.I = \frac{1}{D-2D'-1}\left[\frac{1}{D-2D'} e^{3X+4Y}\right]$$

$$= \frac{1}{D-2D'-1}\left[\frac{1}{3-8}\int e^u du\right] \text{ when } 3X+4Y=u$$

$$= \frac{1}{D-2D'-1}\left[\frac{-1}{5} e^{3X+4Y}\right]$$

$$= \frac{-1}{5}\left[\frac{1}{D-2D'-1} e^{3X+4Y}\right]$$

$$= -\frac{1}{5}\left[\frac{1}{3-8-1} e^{3X+4Y}\right] = \frac{1}{30} e^{3X+4Y}$$

$$= \frac{1}{30} x^3 y^4$$

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Hence the complete soln is

$$z = CF + P.I$$

$$= g_1(y+x^2) + x\,g_2(y+x^2) + \frac{1}{30}x^3y^4$$

where g_1 & g_2 are arbitrary functions.