Q3 Solve the following diff eqn.

(a) $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

Soln Lagrange's eqn.

$$\frac{\partial x}{x^2 - yz} = \frac{\partial y}{y^2 - zx} = \frac{\partial z}{z^2 - xy}$$

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$$\therefore \frac{\partial x-\partial y}{(x-y)(x+y+z)} = \frac{\partial y-\partial z}{(y-z)(x+y+z)} = \frac{\partial z-\partial x}{(z-x)(x+y+z)}$$

Taking the first two members, we have,

$$\frac{\partial x-dy}{x-y} = \frac{\partial y-dz}{y-z}$$

which on integration gives

$$\log(x-y) = \log(y-z) + \log a$$

$$\log\left(\frac{x-y}{y-z}\right) = \log a \text{ or } \frac{x-y}{y-z} = a \quad — (1)$$

Similarly, taking the last two members, we obtain,

$$\frac{y-z}{z-x} = b \quad — (2)$$

from eq (1) & (2), the general solⁿ is

$$\phi\left(\frac{x-y}{y-z}, \frac{y-z}{z-x}\right) = 0$$

(b) $y^2 \frac{z}{x} p + xzq = y^2$

Solⁿ Rewriting the given eqⁿ.

$$y^2zp + x^2zq = y^2x$$

The subsidiary eqⁿ are

$$\frac{\partial x}{y^2z} = \frac{\partial y}{x^2z} = \frac{\partial z}{y^2x}$$

The first two fraction gives $x^2\partial x = y^2dy$.

integrating we get $x^3 - y^3 = a$

again the first & third fractions give

$$xdx = z\partial z$$

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Integrating, we get $x^2 - z^2 = b$ — ②
Hence from eqn ① & ② the complete
Soln is
$$x^3 - y^3 = f(x^2 - z^2).$$