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Q1 $x^2 r - y^2 t + px + qy = \log x$

Sol^n let $x = e^X$, $y = e^Y$ so that $X = \log x$ & $Y = \log y$ & let $D = \frac{\partial}{\partial X}$ & $D' = \frac{\partial}{\partial Y}$

then the given eqn reduced to

$$[D(D-1) - D'(D'-1) + D - D']z = X \quad (1)$$

$$(D^2 - D'^2)z = X$$

Which is a homogeneous linear partial differential equation, with constant coefficients.

$$\therefore CF = \phi_1(Y+X) + \phi_2(Y-X)$$

$$P.I = \frac{1}{D^2 - D'^2}(X) = \frac{1}{(1)^2-(0)^2}\iint u\,du\,du.$$

Where $X = u$.

$$= \int \frac{u^2}{2}du = \frac{u^3}{6} = \frac{X^3}{6}$$

Here solution to eqn (1) is

$$z = \phi_1(Y+X) + \phi_2(Y-X) + \frac{X^3}{6}$$

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$$= \phi_1(\log y + \log x) + \phi_2(\log y - \log x) + \frac{(\log x)^3}{6}$$

Therefore the complete solution to the given differential equation is

$$z = \phi_1(\log xy) + \phi_2\left(\log \frac{y}{x}\right) + \frac{1}{6}(\log x)^3$$

$$z = f_1(xy) + f_2\left(\frac{y}{x}\right) + \frac{1}{6}(\log x)^3$$

Where f_1 & f_2 are arbitrary functions.