$$b_n \sinh\left(\frac{n\pi a}{b}\right) = \frac{2}{b}\int_0^b f(y)\sin\left(\frac{n\pi y}{b}\right)dy$$

$$b_n = \frac{2}{b\sinh\left(\frac{n\pi a}{b}\right)}\int_0^b f(y)\sin\left(\frac{n\pi y}{b}\right)dy \quad (4)$$

Thus $u = \sum_{n=1}^{\infty} b_n \sinh\left(\frac{n\pi x}{b}\right)\sin\left(\frac{n\pi y}{b}\right)$

where b_n is given eq 4.

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shot on moto x4
Akash Morya

$C_3 + C_4 = 0$

$C_4 = -C_3$

and $Y = 0$ at $y = b$

$0 = C_3 e^{kb} - C_3 e^{-kx}$

$C_3 (e^{kb} - e^{-kb}) = 0$

$C_3 = 0, C_4 = 0, y = 0$ (not possible)

Case (VI) $\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = -\frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = -k^2$ (say)

$X = C_1 e^{kx} + C_2 e^{-kx}$,

$Y = C_3 \cos ky + C_4 \sin ky$

At $y = 0, Y = 0, C_3 = 0$

$Y = C_4 \sin ky$

$y = b, Y = 0$

$0 = C_4 \sin ky$

$y = b, Y = 0$

$0 = C_4 \sin kb$

$\sin kb = 0$

$kb = n\pi$

$k = \frac{n\pi}{b}$

Thus $u = (C_1 e^{kx} + C_2 e^{-kx}) C_4 \sin \frac{n\pi y}{b}$ — (2)

$x = 0, u = 0$

$0 = (C_1 + C_2) C_4 \sin \frac{n\pi y}{b}$

$C_1 + C_2 = 0$

$\boxed{C_2 = -C_1}$

from (2)

$u = \frac{2}{2} C_4 C_1 \left(e^{kx} - e^{-kx}\right) \sin \frac{n\pi y}{b}$

$u = \sum_{n=0}^{\infty} b_n \left(\frac{e^{\frac{n\pi x}{b}} - e^{\frac{-n\pi a}{b}}}{2}\right) \sin\left(\frac{n\pi y}{b}\right)$

$f(y) = \sum_{n=0}^{\infty} b_n \sinh\left(\frac{n\pi a}{b}\right) \left(\sin\left(\frac{n\pi y}{b}\right)\right.$

Sec 1

Ans ① Given Laplace equation is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{——} (1)$$

Let $u = XY$, where X is a function of x only and Y is a function of y only.

$$\frac{\partial^2 u}{\partial x^2} = Y \frac{\partial^2 X}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial y^2} = X \frac{\partial^2 Y}{\partial y^2}$$

and from eq (1)

$$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} = 0$$

$$-\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2}$$

Case i : $-\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = 0$ (say)

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = 0$$

and $\frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = 0$

$$X = c_1 x + c_2 \quad Y = c_3 y + c_4$$

At $y = 0$; $Y = 0 \Rightarrow c_4 = 0$

Also $y = b$, $Y = 0 \Rightarrow c_3 = 0$

$Y = 0$

$u = XY = X(0)$

$u = 0$ (not possible)

Case (ii) $-\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = k^2$ (say)

$$\frac{\partial^2 X}{\partial x^2} + k^2 X = 0$$

$$\frac{\partial^2 Y}{\partial y^2} - k^2 Y = 0$$

If $X = c_1 \cos kx + c_2 \sin kx$, $Y = c_3 e^{ky} + c_4 e^{-}$

$y = 0$, $Y = 0$