Section-4

Ans-2

(a) (i) Number of bit strings of length eight that start with a 1 bit: $2^7 = 128$

(ii) Number of bit strings of length eight that end with bits 00: $2^6 = 64$.

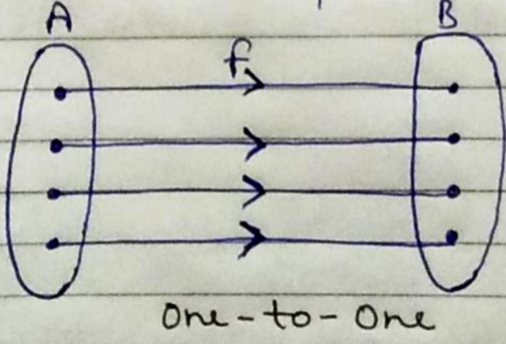
(iii) Number of bit strings of length eight that start with a 1 bit and end with bits 00: $2^5 = 32$
$2^5 = 32$

Hence, the number is $128 + 64 - 32 = 160$.

(b)

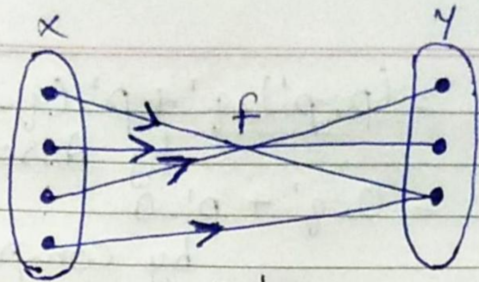
(i) One-to-one function: Let $f: X \to Y$ then f is called one-to-one function if for distinct elements of X there are distinct image in Y i.e., f is one-to-one iff

$$f(x_1) = f(x_2) \text{ implies } x_1 = x_2 \ \forall\, x_1, x_2 \in X$$



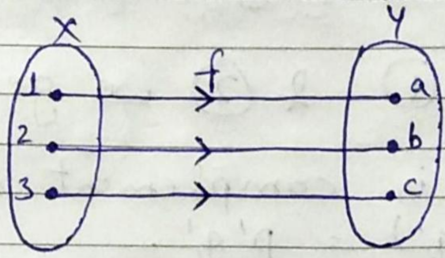
One-to-one

(ii) Onto function: Let $f: X \to Y$ then f is called onto function iff for every element $y \in Y$ there is an element $x \in X$ with $f(x) = y$ or f is onto if Range $(f) = Y$.



Onto

(iii) One-to-One onto function: A function which is both one-to-one and onto is called one-to-one onto function.



One-to-One onto

(C) To prove: $(p+q)' = p'q'$

To prove the theorem we will show that

$$(p+q) + p'.q' = 1$$

Consider $(p+q) + p'.q' = \{(p+q)+p'\}.\{(p+q)+q'\}$ by distributive law

$= \{(q+p)+p'\}.\{(p+q)+q'\}$ by commutative law

$= \{q+(p+p')\}.\{p+(q+q')\}$ by Associative law

$= (q+1).(p+1)$ by complement law

$= 1.1$ by dominance law

$= 1$ ——— (1)

Also Consider

$(p+q).p'q' = p'q'.(p+q)$ by Commutative law

$= p'q'.p + p'q'.q$ by Distributive law

$= p.(p'q') + p'.(q'q)$ by Commutative law

$= (p.p').q' + p'.(q.q')$ by Associative law

$= 0.q' + p'.0$ by complement law

$= q'.0 + p'.0$ by commutative law

$= 0+0$ by dominance law

$= 0$ ——— (2)

From (1) & (2) we get,

$p'q'$ is complement of $(p+q)$ i.e.,
$(p+q') = p'q'$.