# Section-4

**Ans-1**

(a) We prove this by induction on $n$.
Base case: For $n=1$, $11^{n+1}+12^{2n-1} = 11^2+12^1$ $=133$ which is divisible by 133.
Inductive step: Assume that the hypothesis holds for $n=k$, i.e., $11^{k+1}+12^{2k-1}$ $=133A$ for some integer $A$. Then for $n=k+1$,

$$
\begin{aligned}
11^{n+1}+12^{2n-1} &= 11^{k+1+1}+12^{2(k+1)-1} \\
&= 11^{k+2}+12^{2k+1} \\
&= 11 * 11^{k+1} + 144 * 12^{2k-1} \\
&= 11 * 11^{k+1} + 11 * 12^{2k-1} + 133 * 12^{2k-1} \\
&= 11\left[11^{k+1}+12^{2k-1}\right] + 133 * 12^{2k-1} \\
&= 11 * 133A + 133 * 12^{2k-1} \\
&= 133\left[11A + 12^{2k-1}\right]
\end{aligned}
$$

Thus if the hypothesis holds for $n=k$ it also holds for $n=k+1$. Therefore, the statement given in the equation is true.

(b)
(i) Let $H$ be any sub-group of order $m$ of a finite group $G$ of order $n$. Let us consider the left coset decomposition of $G$ relative to $H$.
(ii) We will show that each coset $aH$ consists of $m$ different elements.

Let $H = \{h_1, h_2, \ldots\ldots, h_m\}$

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(iii) Then $ah_1, ah_2, \ldots, ah_m$, are the members of $aH$, all distinct.
For, we have $ah_i = ah_j \Rightarrow h_i = h_j$ by cancellation law in $G$.

(iv) Since $G$ is a finite group, the number of distinct left cosets will also be finite, say $k$. Hence the total no. of elements of all cosets is $km$ which is equal to the total no. of elements of $G$.
Hence $n = mk$
This show that $m$, the order of $H$, is a divisor of $n$, the order of the group $G$.
We also find that the index $k$ is also a divisor of the order of the group.