Section-3

Ans 1

(a) The composition table of G is

*	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	-1	1

(i) Closure property: Since all the entries of the composition table are the elements of the given set, the set G is closed under multiplication.

(ii) Associativity: The elements of G are complex numbers, and we know that multiplication of complex no. is associative.

(iii) Identity: Here, 1 is the identity element.

(iv) Inverse: From the composition table, we see that the inverse elements of 1, -1, i, -i, are 1, -1, -i respectively.

(V) Commutativity: The corresponding rows and columns of the table are identical. Therefore the binary operation is commutative. Hence, (G, *) is an abelian group.

Ans-1

Ring:

(b) A ring $(R,+,.)$ is a set R together with two binary operation + (Addition) and . (Multiplication) defined on R such that the following axioms are satisfied:

(R_1) $(a+b)+c = a+(b+c)$ for all $a,b,c \in R$

(R_2) $a+b = b+a$ for all $a,b \in R$.

(R_3) there exists an element 0 in R such that $a+0 = a$ for all $a \in R$.

(R_4) for all $a \in R$, there exists an element $-a \in R$ such that $a+(-a)=0$.

(R_5) $(a.b).c = a.(b.c)$ for all $a,b,c \in R$.

(R_6) $a.(b+c) = (a.b)+(a.c)$ for all $a,b,c \in R$ (Left distributive law)

(R_7) $(b+c).a = (b.a)+(c.a)$ for all $a,b,c \in R$ (Right distributive law)

Example of commutative ring:

Let $a,b \in R$ $(a+b)^2 = (a+b)$

$\Rightarrow (a+b)(a+b) = (a+b)$

$(a+b)a + (a+b)b = (a+b)$

$(a^2+ba)+(ab+b^2) = (a+b)$

$(a+ba)+(ab+b) = (a+b)$ $\quad (\because a^2=a \;\&\; b^2=b)$

$(a+b)+(ba+ab) = (a+b)+0$

$\Rightarrow ba+ab = 0$

$a+b = 0 \Rightarrow a+b = a+a$ [being every element of its own additive inverse]

$\Rightarrow b = a$

$\Rightarrow ab = ba$

$\therefore$ R is commutative ring.

Example of non-commutative ring :
Consider the set R of 2×2 matrix with real element. For $A, B, C \in R$

$$A * (B + C) = (A * B) + (A * C)$$

also, $(A + B) * C = (A * C) + (B * C)$

$\therefore *$ is distributive over +

$\therefore (R, +, *)$ is a ring.

We known that $AB \neq BA$, Hence $(R, +, *)$ is non-commutative ring.