Section-2

Ans 2

1. Let $G(x) = \sum_{n=0}^{\infty} a_n x^n$ be the generating function of the sequence $a_0, a_1, a_2, \ldots$ We sum both sides of the last eq. starting with $n=1$. To Find that

$$G(x) - 1 = \sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} (8a_{n-1} x^n + 10^{n-1} x^n)$$

$$= 8 \sum_{n=1}^{\infty} a_{n-1} x^n + \sum_{n=1}^{\infty} 10^{n-1} x^n$$

$$= 8x \sum_{n=1}^{\infty} a_{n-1} x^{n-1} + x \sum_{n=1}^{\infty} 10^{n-1} x^{n-1}$$

$$= 8x \sum_{n=0}^{\infty} a_n x^n + x \sum_{n=0}^{\infty} 10^n x^n$$

$$= 8x G(x) + x/(1-10x)$$

Therefore, we have $G(x) - 1 = 8x G(x) + x/(1-10x)$

Expanding the right hand side of the equation into partial fractions gives

$$G(x) = \frac{1}{2}\left(\frac{1}{1-8x} + \frac{1}{1-10x}\right)$$

This is equivalent to $G(x) = \frac{1}{2}\left(\sum_{n=0}^{\infty} 8^n x^n + \sum_{n=0}^{\infty} 10^n x^n\right)$

$$= \sum_{n=0}^{\infty} \frac{1}{2}(8^n + 10^n) x^n.$$

$$a_n = \frac{1}{2}(8^n + 10^n)$$

PAGE NO
DATE / /

Ans-2

2. (i) The compound proposition will be:

$(p \wedge q \wedge r) \Leftrightarrow s$

(ii) Let p be the proposition "It is sunny this afternoon", q be the proposition "It is colder than yesterday", r be the proposition "We will go swimming," s be the proposition, "We will take a canoe trip", and t be the proposition "We will be home by sunset".

Then the hypothesis becomes

$\neg p \wedge q,\ r \rightarrow p,\ \neg r \rightarrow s$, and $s \rightarrow t$.

The conclusion is simply t.

We construct an argument to show that our hypothesis lead to the conclusion as follows:

S. No.	Step	Reason
1.	$\neg p \wedge q$	Hypothesis
2.	$\neg p$	Simplification using step 1
3.	$r \rightarrow p$	Hypothesis
4.	$\neg r$	Modus tollens using steps 2 & 3
5.	$\neg r \rightarrow s$	Hypothesis
6.	s	Modus ponens using steps 4 & 5
7.	$s \rightarrow t$	Hypothesis
8.	t	Modus ponens using steps 6 & 7