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Section-5

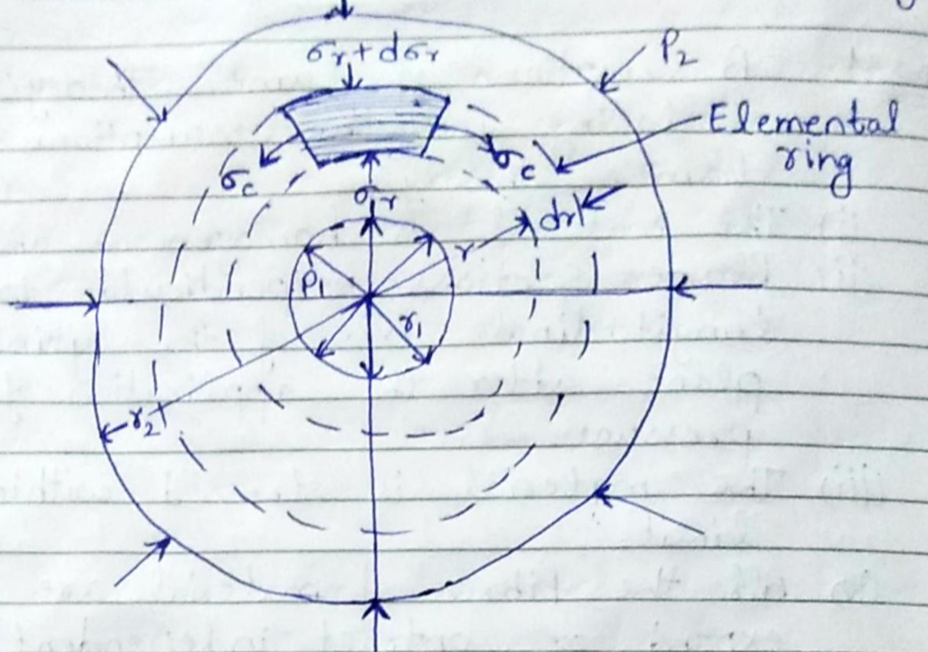
Ans-1 **Assumptions of Lame's Theory:**
Following are the assumptions made in Lame's theory:

(i) The material is homogeneous and isotropic.
(ii) Plane sections perpendicular to the longitudinal axis of the cylinder remain plane after the application of internal pressure.
(iii) The material is stressed within elastic limit.
(iv) All the fibres of material are free to expand or contract independently without being constrained by adjacent fibres.

Derivation:
(i) Consider a thick cylinder subject to internal and external radial stress is shown in fig. Consider an element ring of internal radius r and thickness dr.
(ii) Let,

r_1 = Internal radius of the cylinder.
r_2 = External radius of the cylinder.
l = Length of the cylinder.
p_1 = Pressure on the inner surface of the cylinder.
p_2 = Pressure on the outer surface of the cylinder
σ_r = Internal radial stress on the elemental ring.
$(\sigma_r + d\sigma_r)$ = External radial stress on the elemental ring.

σ_r = Circumferential stress on elemental ring



(iii) The conditions for equilibrium on one half of the elemental ring are as follows:

$$\text{Bursting force} = (\sigma_r \times 2rl) - [(\sigma_r + d\sigma_r) \times 2(r+dr)l]$$
$$= 2l[-\sigma_r dr - r d\sigma_r - dr\, d\sigma_r]$$
$$= -2l(\sigma_r dr + r\, d\sigma_r)$$

(Neglecting the product of small quantities)

$$\text{Resisting force} = 2\sigma_c l\, dr$$

(iv) Equating the resisting force to bursting force we get

$$2\sigma_c l\, dr = -2l(\sigma_r dr + r\, d\sigma_r)$$
$$\sigma_c = -\sigma_r - r\frac{d\sigma_r}{dr} \qquad \text{—— (1)}$$

(v) Now let us obtain another relation between the radial stress and circumferential stress by using the conditional that the longitudinal strain (ε_1) at any point in the section is same.

The longitudinal stress,

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$$\sigma_1 = \frac{p_1 \times \pi r_1^2}{\pi(r_2^2 - r_1^2)} = \frac{p_1 r_1^2}{(r_2^2 - r_1^2)}$$

(vi) Hence at any point in the section of the element ring considered above the following three principal stresses exist,

a) The radial stress (pressure), σ_r.

b) The circumferential stress, σ_c.

c) The longitudinal tensile stress, σ_l.

(vii) Since the longitudinal strain (ε_1) is constant we have,

$$\varepsilon_l = \frac{\sigma_1}{E} - \frac{\mu \sigma_c}{E} + \frac{\mu \sigma_r}{E} = \text{Constant}$$

But, since σ_l, μ & E are constant

$$\therefore \quad \sigma_c - \sigma_r = \text{Constant}$$

Let, $\quad \sigma_c - \sigma_r = 2a \quad$ — (2)

Putting $\sigma_c = (\sigma_r + 2a)$ in eq (1) we get.

$$(\sigma_r + 2a) = -\sigma_r - r\frac{d\sigma_r}{dr}, \quad \frac{d\sigma_r}{dr} = \frac{-2(\sigma_r + a)}{r}$$

$$\frac{d\sigma_r}{\sigma_r + a} = -\frac{2dr}{r} \quad \text{— (3)}$$

Integrating both sides we get

$$\log_e(\sigma_r + a) = -2\log_e r + \log_e b$$

$$(\because \log_e b = \text{Constant})$$

$$\therefore \log_e(\sigma_r + a) = \log_e \frac{b}{r^2}$$

$$\sigma_r = \frac{b}{r^2} - a \quad \text{— (4)}$$

$$\sigma_c = \frac{b}{r^2} + a \quad \text{— (5)}$$

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(viii) Similarly,

(ix) The eq. (4) & (5) are called Lame's eq.

(x) The constant a & b can be evaluated from the known internal and external radial pressure and radius.

(xi) It may be noted that in the above eq σ_r is compressive and σ_c is tensile.