

Section-3

Q1 Ring - A non empty set is a ring if it is equipped with two binary operations called addition and multiplication & denoted by '+' and '·' respectively i.e for all $a, b \in R$ we have $a+b \in R$ and $a \cdot b \in R$ and it satisfies the following properties.

i) Addition is associative i.e
 $(a+b)+c = a+(b+c) \forall a, b, c \in R$

ii) Addition is commutative
 $a+b = b+a \forall a, b \in R$

iii) There exists an element $0 \in R$ such that
 $0+a = a = a+0, \forall a \in R$

To each element a in R there exists an element $-a$ in R such that $a+(-a) = 0$

v) Multiplication is associative i.e.

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c, \forall a, b, c \in R$$

vi) Multiplication is distributive with respect to addition i.e for all $a, b, c \in R$

Example of ring with zero divisors
 $R =$ (a set of 2×2 matrices)

Field \Rightarrow A ring R with at least two elements is called a field if it has following properties

i) R is commutative

R has unity

R is such that each non-zero element possesses multiplicative inverse

For ~~ex~~ example - The ring of real numbers and complex numbers are also fields.