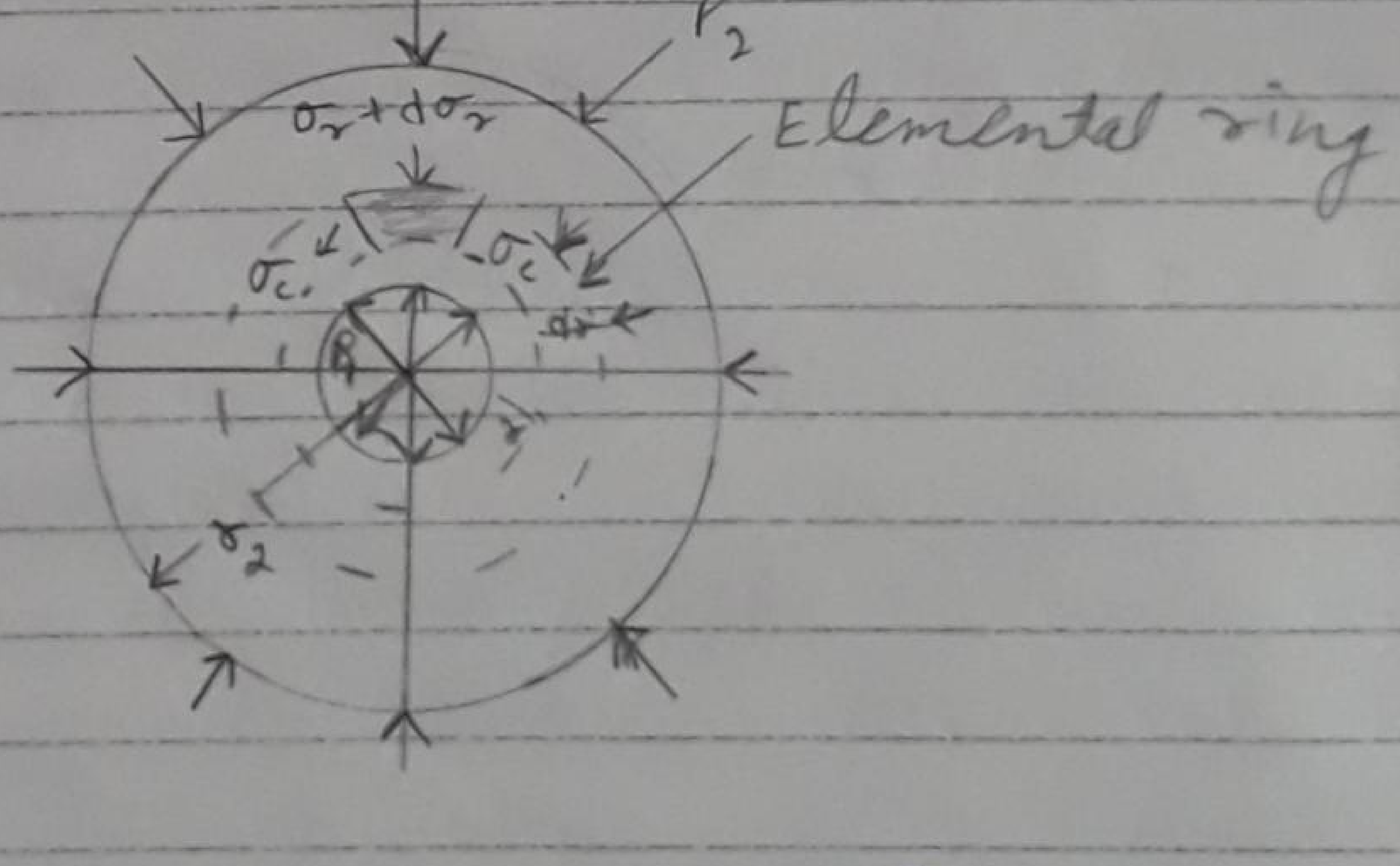
Section - 5

Q1 Derivation -

i) Consider a thick cylinder subject to internal & external radial stress. Consider an element ring of internal radius r and thickness dr

ii) Let r_1 - Internal radius of the cylinder
r_2 - External radius of the cylinder.
l = Length of the cylinder.
P_1 - Pressure on the inner surface of the cylinder.
P_2 - Pressure on the outer surface of the cylinder.
σ_r = Internal radial stress on the elemental ring.
$(\sigma_r + d\sigma_r)$ = External radial stress on the elemental ring.
σ_c = Circumferential stress on elemental ring.



The conditions for equilibrium on one half of the elemental ring to those in the case of thin cylinder.

Bursting force $= (\sigma_r \times 2rl) - [(\sigma_r + d\sigma_r) \times 2(r+dr)l]$
$= 2l[-\sigma_r dr - r d\sigma_r - dr\, d\sigma_r] = -2l$
$(\sigma_r dr + r d\sigma_r)$

Resisting force $= 2\sigma_c\, l\, dr$

Equating the resisting force to bursting force

$$2\sigma_c l dr = -2l(\sigma_r dr + r d\sigma_r)$$

$$\sigma_c = -\sigma_r - \frac{r\, d\sigma_r}{dr}$$

$$\sigma_l = \frac{P_1 \times \pi r_1^2}{\pi(r_2^2 - r_1^2)} = \frac{P_1 r_1^2}{(r_2^2 - r_1^2)}$$

Following three principal stress exist

σ_r, σ_c, σ_l

$$\varepsilon_l = \frac{\sigma_l}{E} = \frac{u\sigma_c}{E} + \frac{u_1 \sigma_r}{E}$$

$\sigma_c - \sigma_r =$ constant

Let $\sigma_c - \sigma_r = 2a$

$$\sigma_r + 2a = -\sigma_r - \frac{r d\sigma_r}{dr},\ \frac{d\sigma_r}{dr} = \frac{-2(\sigma_r + a)}{r}$$

$$\frac{d\sigma_r}{\sigma_r + a} = -\frac{2dr}{r}$$

$$\log(\sigma_r + a) = -2\log_e r + \log_e b$$

$$\log_e(\sigma_r + a) = \log_e \frac{b}{r^2}$$

$$\sigma_r = \frac{b}{r^2} - a$$

$$\sigma_c = \frac{b}{r^2} + a$$

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